Deterioration Forecasting Model with Multistage Weibull Hazard Functions

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Abstract: In this paper, a time-dependent deterioration forecasting model is presented. In the model the deterioration process is described by transition probabilities, which are conditional upon actual in-service duration. The model is formulated by the multistage Weibull hazard model defined by using multiple Weibull hazard functions. The model can be estimated based upon inspection data that are obtained at discrete points in time. The applicability of the model and the estimation methodology presented in this paper are investigated against an empirical data set of highway utilities in the real world.

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Introduction

Effective management of any infrastructure utilities such as tunnel lighting in highway systems requires comprehensive understanding of the entire operational processes of the utility as well as monitoring of its performance and conditions throughout its operational life. Continuous inspection and monitoring of the system are, however, often technically or financially difficult. Therefore, a need to develop an analytical deterioration forecasting model that can estimate the deterioration speed of either an individual component or the entire infrastructure system has been widely recognized.

Various studies have attempted incorporation of historical background of infrastructure performance into a deterioration model. For example, Aoki et al. (2007) proposed the Weibull distribution function to estimate the deterioration of lighting reflectors in tunnel systems. This expressed the condition state of tunnel lighting reflectors in binary terms. However, it is known that the actual deterioration process of most infrastructure systems is better described by plural discrete condition states (Shahin 2005). In order to overcome this limitation, the Markovian transition probability can be used to express two or more condition states in the deterioration process of infrastructure.

The Markov chain model is a stochastic approach that is widely used to forecast the deterioration speed of an infrastructure system such as a bridge network (Madanat et al. 1995; Guido et al. 2004; Morcous 2006; Robelin and Madanat 2007). Tsuda et al. (2006) further improved the Markov chain model by proposing a handy methodology to estimate the Markovian transition probability. The advantages of these models are that they predict future deterioration according to information from two inspection times and they do not require extensive historical data.

The present paper proposes a new deterioration forecasting model for infrastructure management, which expresses the deterioration speed in two or more condition states in conjunction with elapsed time and follows the Weibull distribution function. To begin with, a brief review of the background literature is provided. "Formulation of the Model" and "Estimation Method" detail the mathematical formulation of the time-dependent transition probability using the Weibull distribution function and the estimation approach. "Empirical Analysis" presents an empirical study using actual data from a tunnel lighting system in Japan. Finally, "Conclusions" summarizes the contributions made by this paper, and points out future research needs.

Research Background

Outline of Past Research

In the field of infrastructure management, various models on deterioration forecasting have been widely documented. One major feature of the models is to simulate the deterioration process. In addition, the models can be used for setting up maintenance and repair strategies as well as proposing life cycle cost analysis. Especially under the requirements of infrastructure management at the network level, these objectives are particularly imperative (Aoki et al. 2007; Tsuda et al. 2006).

Past research has paid much attention to the physical mechanism of the deterioration of structures (Mishalani and Koutsopoulos 1995; Mishalani and Madanat 2002; Steven and Laszlo 2000). However, past research, because it did not clearly specify the statistical estimation method being used for analysis, remained at

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a rudimentary stage of development, with several problems from the estimation results emerging as limitations. Moreover, a great deal of inspection data are generally required to ensure the accuracy of the models.

In recent decades, studies aiming toward statistical application have been extensively recorded (Lancaster 1990; Gourieroux 2000). For instance, Shin and Madanat (2003) proposed employing the Weibull deterioration hazard model to forecast the time when cracks start to appear on pavement structures. In a similar approach, Aoki et al. (2007) empirically verified the effectiveness of applying the Weibull distribution function to forecast the deterioration of tunnel lighting reflectors. However, as earlier mentioned, these models portrayed deterioration progress only by using a binary condition state, which did not totally reflect the actual plural condition states applied in infrastructure management.

Methods of tackling emerging problems have been proposed. A typical example is the multistage model developed by Lancaster (1990) for the behavior of labor transition, in which the writer described a rational approach to estimating the transition probability from multiple condition states. The mechanism in the multistage model is that the condition state changes from one state to other states only in one-step. This boundary creates problems in its application to infrastructure management since condition state transitions are often observed in more than one-step changes. In an effort to overcome this limitation, Tsuda et al. (2006) described the vertical transitive relation between condition states and proposed a method to estimate Markov transition probability according to the multistage hazard model for bridge management.

The Markov hazard model proposed by Tsuda et al. (2006) has wide applicability into many fields. However, the Markov transition probability is characterized by the fact that the deterioration process does not depend on past deterioration history. Additionally, there is no concrete guarantee that the deterioration process genuinely satisfies the Markov properties, especially in cases when the total operation duration of infrastructure is taken into estimation. This limitation has generated a motivation for the development of this paper, which considers multistate transition between condition states and historical operation time.

Deterioration Process and Condition States

In order to analyze and forecast the deterioration of infrastructure components, it is necessary to accumulate time series data on the condition states of the components. The historical deterioration process of an infrastructure component is described in Fig. 1. This figure shows the deterioration progress of a component that has not been repaired. In reality, there exists uncertainty in the deterioration progress of the component and moreover, the condition state at each point in the time axis is restricted to the time at which visual inspection is carried out.

In Fig. 1, τ represents real calendar time. The deterioration of the infrastructure starts immediately after its opening to the public at initial time τ_0. The condition state of a component is expressed by rank I representing a condition state variable $i(i=1,\ldots,I)$. For a component in a good or new situation, its condition state is given as $i=1$. The increasing of condition state i describes progressing deterioration. A value of $i=I$ indicates that a component has reached its service limit (absorbing state). In Fig. 1, for each discrete time $\tau_i(i=1,\ldots,I-1)$ on the horizontal time axis, we can

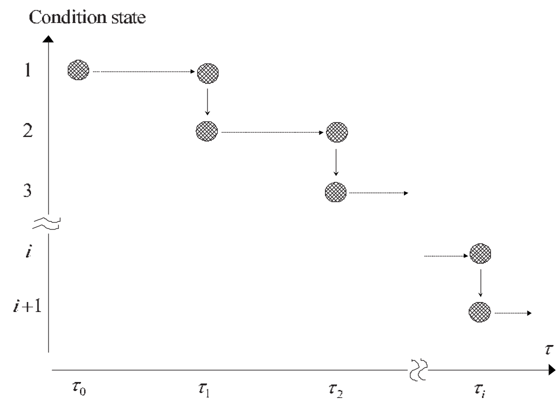

Fig. 1. Transitions among the condition states

observe the condition state increasing from i to $i+1$. Hereinafter, τ_i refers to the time at which the transition from condition state i to $i+1$ occurs.

Information regarding the deterioration process of infrastructure can be acquired through periodical visual inspections. In fact, however, continuous monitoring and inspection of systems are often technically or financially difficult. Thus, normal practice is to carry out discrete periodical visual inspections throughout the service life of the infrastructure. The model assumes that initial inspection is carried out at time τ_A, when the condition state observed by inspection is $i(i=1,\ldots,I-1)$. Future deterioration progresses in an uncertain manner, with many possible deterioration paths. Nevertheless, among the infinite set of possible deterioration paths, only one is finally realized.

Fig. 2 shows four possible sample deterioration paths. Path 1 shows no transition in the Condition State 1 from initial time τ_0 to first inspection time τ_A. In Paths 2 and 3, the condition state has advanced to Condition State 2 at times τ_1^2 and τ_1^3, respectively. However, Condition State 2 can be realized only at inspection time τ_A. The exact times of τ_1^2 and τ_1^3, at which point the real change between condition states happens, cannot be precisely captured. Similarly, in the case of Path 4, the times τ_1^4 and τ_2^4, at which, Condition State 2 and 3, respectively occur, cannot be defined.

Deterioration State Probability

We denote s as an arbitrary elapsed time counted from the initial time τ_0. The state variable $h(s)$ expresses the actual condition

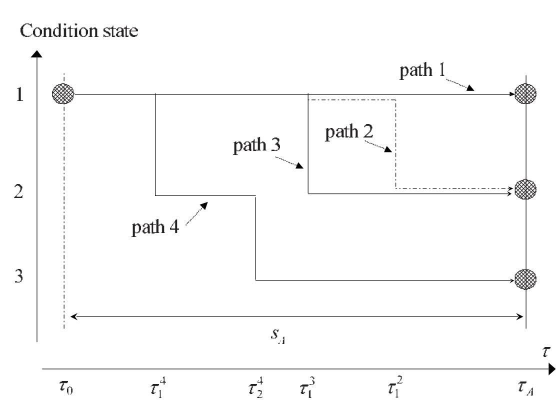

Fig. 2. Transition pattern of the condition state

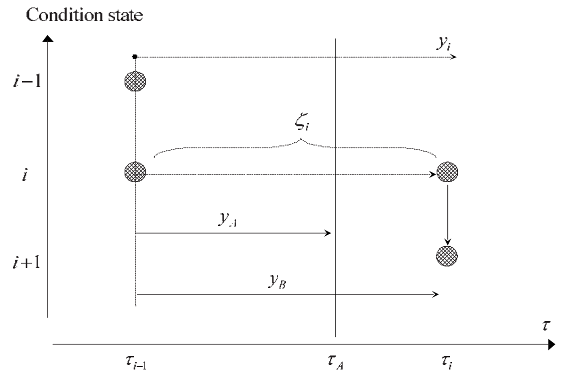

Fig. 3. Modeling of deterioration process

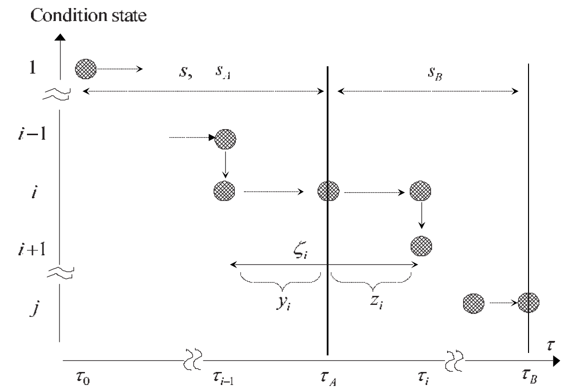

Fig. 4. Deterioration process from initial time and observation of condition state

state corresponding to time $\tau=\tau_0+s$. The deterioration process is described by using conditional probability, which describes condition state $h(s)=i$ as occurring at time s dependent on the given condition state at τ_0 (hereafter referred to deterioration state probability)

$$\text{Prob}[h(s)=i|h(0)=1]=\pi_i(s) \tag{1}$$

If the deterioration state probability π_i is defined in the range of condition state $i(i=1,\ldots,I)$, then a time-dependent deterioration state probability vector can be further expressed as

$$\Pi(s)=\begin{bmatrix}\pi_1(s)\\ \vdots \\ \pi_I(s)\end{bmatrix} \tag{2}$$

The deterioration state probability in Eq. (1) represents the probability of each condition state i being observed at time $\tau=\tau_0+s$. In other words, it expresses the probability of state occurrence in the elapsed time s from the initial time. The summation $\Sigma_{i=1}^{I}\pi_i(s)=1$ is justified by the definition of deterioration state probability.

Formulation of the Model

Weibull Hazard Model

This section discusses the mathematical expressions for Eq. (1) by applying the Weibull distribution function. Also, described in this section is the methodology for estimating the deterioration state probability by using information from collected individual data. Fundamental background knowledge on hazard analysis can be found in Lancaster (1990) and Gourieroux (2000). In the next section, we briefly explain the assumption of the deterioration process, to which Figs. 3 and 4 refer.

In Fig. 3, condition state $i-1$ changes to i at time τ_{i-1}. We define ζ_i as the life span of condition state i and y_i as the elapsed time being counted from $y_i=0$ at initial time τ_{i-1}. Similarly, duration $y_A=\tau_A-\tau_{i-1}$ is understood as elapsed time between τ_{i-1} and τ_A. By employing the Weibull distribution function, the survival probability function $\tilde{F}_i(y_i)$ and the probability density function $f_i(\zeta_i)$, which describes the deterioration process, are given in following equations:

$$\tilde{F}_i(y_i)=\exp(-\theta_i y_i^{\alpha_i}) \tag{3}$$

$$f_i(\zeta_i)=\theta_i\alpha_i\zeta_i^{\alpha_i-1}\exp(-\theta_i\zeta_i^{\alpha_i}) \tag{4}$$

Deterioration State Probability from Initial Time

We assume the opening of an infrastructure facility at time τ_0 with Condition State 1 (Fig. 2). At time τ, the observed condition state is i. On the horizontal time axis, condition states of the infrastructure facility can be displayed with respect to arbitrary time from τ_0 to τ. The probability of the event that Condition State 1 changes to Condition State i can be represented by state probability $\pi_i(s)$ (where $s=\tau-\tau_0$).

$i=1$

Condition state remains as one until time τ. The deterioration state probability $\pi_1(s)$ is exactly equal to the survival probability expressed in Eq. (3)

$$\pi_1(s)=\tilde{F}_1(s)=\exp(-\theta_1 s^{\alpha_1}) \tag{5}$$

$i=2$

In the case when condition state $i=2$ is observed at time τ, the condition state changes from one to two at a time $\tau_1\in[\tau_0,\tau]$. The probability density that the life span of Condition State 1 becomes $\zeta_1=\tau_1-\tau_0$ can be expressed as $f_1(\zeta_1)$ by using the Weibull function. $\zeta_1(\geq 0)$ is a random variable, which owns its value in the following range:

$$0\leq\zeta_1<s \tag{6}$$

State probability $\pi_2(s)$ with condition state $i=2$ being observed at time τ is shown in the next equation

$$\pi_2(s)=\int_0^s f_1(\zeta_1)\tilde{F}(s-\zeta_1)d\zeta_1 \tag{7}$$

$3\leq i<I$

For a general case, as condition state at time τ can take value between $3\leq i<I$, the event of changes in condition state will occur at respective times $\tau_1,\ldots,\tau_{i-1}(\tau_0\leq\tau_1\leq\cdots\leq\tau_{i-1}<\tau)$. The following steps describe the mechanism of these changes. At first, Condition State 1 remains in a duration from time τ_0 to time $\tau_0+\zeta_1\in[\tau_0,\tau]$, as illustrated in Fig. 4. Second, at time τ_1, condition state changes from 1 to 2. Third, Condition State 2 remains in a duration from time τ_1 until time $\tau_2=\tau_1+\zeta_2\in[\tau_1,\tau]$, before turning into Condition State 3 exactly at time τ_2. Fourthly, after undergoing similar processes, condition state advances to i at time $\tau_{i-1}=\tau_{i-2}+\zeta_{i-1}\in[\tau_{i-2},\tau]$, and remains at condition state i until time τ.

To simulate the occurrence of these events, we use the probability density $q_i(\zeta_1,\ldots,\zeta_{i-1})$ in the entire duration $s=\tau-\tau_0$

$$q_i(\zeta_1,\ldots,\zeta_{i-1})=\left\{\prod_{m=1}^{i-1}f_m(\zeta_m)\right\}\tilde{F}_i\left(s-\sum_{m=1}^{i-1}\zeta_m\right) \tag{8}$$

Random variable $\zeta_m(\geq 0)$ takes its value in the range to satisfy

$$0\leq\zeta_1+\zeta_2+\cdots+\zeta_{i-1}<s \tag{9}$$

Therefore, the state probability $\pi_i(s)$, which represents observed condition state $i(i=3,\ldots,I-1)$ at time $\tau=\tau_0+s$, can be expressed as follows:

$$\begin{aligned}\pi_i(s)&=\int_0^s f_i(\zeta_1)\int_0^{s-\zeta_1}f_i(\zeta_2)\cdots\int_0^{s-\zeta_1-\cdots-\zeta_{i-3}-\zeta_{i-2}}\\&\quad\times f_i(\zeta_{i-1})\tilde{F}_i\left(s-\sum_{m=1}^{i-1}\zeta_m\right)d\zeta_1\cdots d\zeta_{i-1}\\&=\int_0^s\int_0^{s-\zeta_1}\cdots\int_0^{s-\sum_{m=1}^{i-2}\zeta_m}q_i(\zeta_1,\cdots,\zeta_{i-1})d\zeta_1\cdots d\zeta_{i-1}\end{aligned} \tag{10}$$

i=I

Condition state I is absorbing state, which refers to the worst deterioration. At the time when I has been reached, if no repair occurs, the state I will remain forever. From the definition of the deterioration state probability, the probability of observing absorbing state I is shown in the following equation:

$$\pi_I(s)=1-\sum_{m=1}^{I-1}\pi_m(s) \tag{11}$$

Note that in the figure, the initial time is τ_0. Condition state i is observed at time τ_A. For two inspection times τ_A and τ_B, we represent $s_A=\tau_A-\tau_0$, $s_B=\tau_B-\tau_A$ as elapsed time. The time length y_i is measured from time τ_{i-1} to time τ_A, and z_i is measured from time τ_A to time τ_i. The total life span (survival time) of condition i is expressed as $\zeta_i=y_i+z_i$.

Simultaneous Occurrence Probability of Condition State at Two or More Visual Inspection Times

We assume that there are two inspection times τ_A and τ_B, at which the condition states i and j $(i\leq j; i=1,\ldots,I-1)$ are observed, respectively. τ_0 is the initial time of the deterioration process as shown in Fig. 4. The transition pattern of condition states occurs in the following steps. First, at time τ_{i-1}, condition state $i-1$ changes into condition state i. However, condition state i can be revealed only at inspection time τ_A. The duration of this event can therefore be defined as $\tau_A=\tau_{i-1}+y_i$. Second, at time $\tau_i=\tau_A+z_i$, the condition state advances from i to $i+1$. Third, condition state $i+1$ will rise to $j-1$ at time τ_{j-1}. Finally, after τ_{j-1}, the condition state will reach j and remain in condition state j until inspection time τ_B.

In Fig. 4, we define durations $s_A=\tau_A-\tau_0$ and $s_B=\tau_B-\tau_A$. It should be recognized from Fig. 4 that condition state $i-1$ changes into condition state i at time $\tau_{i-1}=\tau_A-y_i$. In other words, condition state i is revealed at inspection time τ_A; however, it has already existed over the duration y_i. If condition state j observed at inspection τ_B is considered, the probability for this event to happen is thus dependent on the information concerning condition state i. Thus, by the definition of conditional probability, the following conditional probability density function is defined:

$$\begin{aligned}&g_{ij}(s_B,z_i,\zeta_{i+1},\ldots,\zeta_{j-1}|y_i)\\&\quad=\frac{f_i(y_i+z_i)}{\tilde{F}_i(y_i)}\prod_{m=i+1}^{j-1}f_m(\zeta_m)\tilde{F}_j\left(s_B-z_i-\sum_{m=i+1}^{j-1}\zeta_m\right)\end{aligned} \tag{12}$$

In Eq. (12), y_i and z_i=durations measured from time τ_{i-1} to time τ_A and from time τ_A to time τ_i, respectively, as shown in Fig. 4. The life span of condition state i is defined by means of variable $\zeta_i=y_i+z_i$. Variables $z_i(\geq 0),\zeta_{i+1}(\geq 0),\ldots,\zeta_{j-1}(\geq 0)$ are random variables with their values to satisfy the following equation:

$$0\leq z_i+\sum_{m=i+1}^{j-1}\zeta_m<s_B \tag{13}$$

Given the elapsed time y_i and condition state i observed at inspection time τ_A, we define the conditional probability $\kappa_{ij}(s_B|y_i)$, to which condition state j is observed at inspection time $\tau_B=\tau_A+s_B$

$$\begin{aligned}\kappa_{ij}(s_B|y_i)&=\int_0^{s_B}\int_0^{s_B-z_i}\cdots\int_0^{s_B-z_i-\sum_{m=i+1}^{j-2}\zeta_m}\\&\quad\times g_{ij}(s_B,z_i,\zeta_{i+1},\ldots,\zeta_{j-1}|y_i)dz_id\zeta_{i+1}\cdots d\zeta_{j-1}\end{aligned} \tag{14}$$

Condition state i can appear at any arbitrary time from the initial time to inspection time τ_A. The duration y_i therefore has a range in the domain $0\leq y_i\leq s_A$. Eventually, we can define the probability density $\eta_i(s_A,y_i)$, which describes the probabilistic relation of condition state i occurring at time $\tau_{i-1}=\tau_A-y_i$

$$\begin{aligned}\eta_i(s_A,y_i)&=\left\{\int_0^{s_A-y_i}f_1(\zeta_1)\int_0^{s_A-y_i-\zeta_1}f_2(\zeta_2)\cdots\right.\\&\quad\left.\int_0^{s_A-y_i-\cdots-\zeta_{i-3}}f_{i-1}(\zeta_{i-1})d\zeta_1\cdots d\zeta_{i-2}\right\}\tilde{F}_i(y_i)\\&=\left\{\int_0^{s_A-y_i}\int_0^{s_A-y_i-\zeta_1}\cdots\int_0^{s_A-y_i-\sum_{m'=1}^{i-3}\zeta_{m'}}\right.\\&\quad\left.\times\prod_{m'=1}^{i-1}f_{m'}(\zeta_{m'})d\zeta_1\cdots d\zeta_{i-2}\right\}\tilde{F}_i(y_i)\end{aligned} \tag{15}$$

$$\zeta_{i-1}=s_A-y_i-\sum_{m'=1}^{i-2}\zeta_{m'} \tag{16}$$

As a sequel, we are able to define the explicit form for transition probability $\pi_{ij}(s_A,s_B)$, which expresses the conditional probability for condition state i being observed at τ_A and condition state j being observed at $\tau_B=\tau_0+s_A+s_B$

$$\begin{aligned}\pi_{ij}(s_A,s_B)&=\text{Prob}[h(s_A)=i,h(s_A+s_B)=j]\\&=\int_0^{s_A}\eta_i(s_A,y_i)\kappa_{ij}(s_B|y_i)dy_i\end{aligned} \tag{17}$$

The probability that condition state I is observed at inspection time τ_A can be seen in Eq. (11). If at inspection τ_B, condition state I is revealed, we can define the following transition probability:

$$\pi_{iI}(s_A,s_B)=\pi_i(s_A)-\sum_{j=i}^{I-1}\pi_{ij}(s_A,s_B) \tag{18}$$

Management Indicator for Infrastructure Management

The life expectancy of condition state i is an important indicator for infrastructure management. Life expectancy is viewed as duration, in which condition state i remains until entering condition state $i+1$. In other words, life expectancy of condition state i is the remaining duration counted from initial time until time τ_i, at which, condition state i changes to condition state $i+1$. Probabilistically, life expectancy of condition state i can be expressed by means of the survival probability function $\tilde{F}_i(y_i)$ (Lancaster 1990)

$$\mathrm{RMD}(i) = \int_0^{\infty} \tilde{F}_i(y_i)dy_i \tag{19}$$

The abbreviation RMD stands for "remaining duration." Based on Eq. (3), we have the following equation:

$$\mathrm{RMD}(i) = \int_0^{\infty} \exp(-\theta_i y_i^{\alpha_i})dy_i \tag{20}$$

Management indicator RMD(i) is estimated based on the assumption that at time τ_{i-1} condition state changes from $i-1$ to i, as shown in Fig. 4. This calculation seems to have the limitation that it does not capture the historical duration measured from initial time. Thus, it is necessary to define the life expectancy of condition state i based on the initial time. We denote RL(i), standing for "remaining life," as a management indicator, which indicates the duration of condition state i counted from initial time. As can be seen from Fig. 4, RL(i) is actually measured from time τ_0 to time τ_i. Given the total duration s for condition state i to remain until reaching condition state $i+1$, we can define the probability density $\rho_i(s)$ for condition state i ending its service life at time $\tau=\tau_0+s$

$$\rho_i(s) = \int_0^{s}\int_0^{s-\zeta_1}\cdots\int_0^{s-\Sigma_{m=1}^{i-2}\zeta_m} \prod_{m=1}^{i-1} f_m(\zeta_m) f_i\left(s-\sum_{m=1}^{i-1}\zeta_m\right)d\zeta_1\cdots d\zeta_{i-1} \tag{21}$$

RL(i)=expected period until the ending of condition state i counted from initial time, and thus can be further defined

$$\mathrm{RL}(i) = \int_0^{\infty} s\rho_i(s)ds \tag{22}$$

It is noted that RMD and RL are fundamentally estimated based on two different assumptions of starting time. Thus, there exists a high possibility that the estimation results of these two management indicators are different. In addition to management indicators RMD(i) and RL(i), there is a need to estimate the life expectancy of condition state j as well. As a matter of fact, the event condition state j appears conditionally dependent on condition state i, which seems to be observed at inspection time τ_A. By the definition of conditional probability, we can define the conditional probability density $\nu_j(s|h(s_A)=i)$, at which condition state j will disappear given the visual observed condition state i at time $\tau_A=\tau_0+s_A$ and the elapsed duration time s

$$\nu_j(s|h(s_A)=i) = \frac{M\times N}{\pi_i(s_A)} \tag{23}$$

where

$$M = \int_0^{s_A}\int_0^{s_A-y_i}\int_0^{s_A-y_i-\zeta_1}\cdots\int_0^{s_A-y_i-\Sigma_{m'=1}^{i-3}\zeta_{m'}} f_i(y_i+z_i) \times \prod_{m'=1}^{i-1} f_{m'}(\zeta_{m'})dy_i d\zeta_1\cdots d\zeta_{i-2}dz_i \tag{24}$$

$$N = \int_0^{s}\int_0^{s-z_i}\cdots\int_0^{s-z_i-\Sigma_{m=i+1}^{j-2}\zeta_m} \times \prod_{m=i+1}^{j-1} f_m(\zeta_m) f_j\left(s-z_i-\sum_{m=i+1}^{j-1}\zeta_m\right)d\zeta_{i+1}\cdots d\zeta_{j-1}$$

$$(i\le j; i,j=1,\ldots,I-1) \tag{25}$$

The denominator of Eq. (23) refers to deterioration state probability for condition state i, which remains until time s_A. In the numerator, M represents the event that condition state i remains until increment time z_i, and N represents the event that condition state i changes to j at elapsed time ζ_{j-1} and stays up to duration s. Eventually, we define the life expectancy of condition state j ($j \ge i$) as $RL_j(h(s_A)=i)$, which conditionally depends on condition state i with duration s_A

$$RL_j(h(s_A)=i) = \int_0^{\infty} s\nu_j(s|h(s_A)=i)ds$$

$$(i\le j; i,j=1,\ldots,I-1) \tag{26}$$

Estimation Method

Content of Data from Visual Inspection

Suppose visual inspection data on the same kind of K infrastructure components is available, an inspection sample $k(k=1,\ldots,K)$ describes a visual inspection time carried out at $\bar{\tau}_A^k$ with the concerning condition state $h(\bar{s}^k)$. The symbol $\lfloor - \rfloor$ indicates an actual measurement. The duration between initial time $\bar{\tau}_0^k$ and the first inspection time $\bar{\tau}_A^k$ is $\bar{s}^k=\bar{\tau}_A^k-\bar{\tau}_0^k$. In addition, a dummy variable $\delta^k=\{\bar{\delta}_i^k(i=1,\ldots,I)\}$ based on the deterioration progress patterns in the duration $\bar{s}^k$ is defined as

$$\bar{\delta}_i^k = \begin{cases} 1 & h(\bar{s}^k)=i \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

Furthermore, in order to describe the information in sample k, we use characteristic vector $\bar{x}^k=(\bar{x}_1^k,\ldots,\bar{x}_N^k)$ and elapsed duration $\bar{s}^k$. $\bar{x}_n^k(n=1,\ldots,N)$ represents the value of a characteristic variable n in the sample k such as structural characteristics, environmental condition and so on. Thus, the information contained in inspection sample k can be rearranged as $\bar{\xi}^k=(\delta^k,\bar{s}^k,\bar{x}^k)$. As a result, we can further express the Weibull hazard function for sample k as

$$\lambda_i^k(y_i) = \theta_i^k\alpha_i y_i^{\alpha_i-1}(i=1,\ldots,I-1) \tag{28}$$

It is noted that the hazard function is not defined for condition state I since I is absorbing state and $\lim_{s\to\infty}\pi_I(s)=1$. As a matter of course, the value of hazard rate $\theta_i^k(i=1,\ldots,I-1;k=1,\ldots,K)$ changes according to the property of characteristic vectors of sample k. The dependency of hazard rate on characteristic vector $\bar{x}^k$ can be formulated by means of functional relationship as

$$\theta_i^k = \bar{x}^k \beta_i' \tag{29}$$

where $\beta_i = (\beta_{i1}, \ldots, \beta_{iN})$ = row vector of unknown parameter $\beta_{in}(n = 1, \ldots, N)$ and the symbol $'$ indicates transposition. The functional relationship between hazard rate and characteristic variable can be changed according to preferences in estimation. This issue can be further viewed in the relationship assumption in our empirical study.

Later in this section, the methodology to estimate the transition probability will be presented. At first, based on the Weibull hazard function $\lambda_i^k(y_i)$ with collected sample information $\bar{\xi}^k(k = 1, \ldots, K)$, the likelihood function for transition probability is defined. Based on the maximum likelihood estimation approach, we can obtain the values for unknown parameters in Eq. (29) and further for the parameterized values of the Weibull hazard function. Second, the estimation method is proposed for the transition probability when there are two or more than two inspection data. Finally, we explain the necessity of estimating the expected deterioration state probability as a representative value when there is a large pool of sampling data.

Estimation of Weibull Hazard Function

As earlier mentioned, data concerning inspection sample k can be rearranged as $\bar{\xi}^k = (\delta^k, \bar{s}^k, \bar{x}^k)$. The application of the Weibull hazard function in estimating the deterioration state probability is discussed in Eqs. (5), (7), (10), and (11). Applying the characteristic vector $\bar{x}^k$ of infrastructure component, we can calculate the hazard rate expressed in Eq. (29). Moreover, the deterioration state probability depends on the duration of operation $\bar{s}^k$ after the opening time of the infrastructure. Therefore, in order to express clearly this characteristic, the deterioration state probability $\pi_i(\bar{s}^k)$ can be defined as a function of measured visual inspection data $(\bar{s}^k, \bar{x}^k)$ and unknown parameter vector $\gamma = \{\alpha, \beta_i(i = 1, \ldots, I-1)\}$. $\alpha = (\alpha_1, \ldots, \alpha_{I-1})$ is a row vector of unknown parameter $\alpha_i(i = 1, \ldots, I-1)$.

If the deterioration progress of the infrastructure components in K samples are assumed to be mutually independent, the log-likelihood function expressing the simultaneous probability density of the deterioration transition pattern for all inspection samples is

$$\ln[L(\gamma)] = \ln\left[\prod_{i=1}^{I}\prod_{k=1}^{K}\{\pi_i(\bar{s}^k, \bar{x}^k : \gamma)\}^{\bar{\delta}_i^k}\right] = \sum_{i=1}^{I}\sum_{k=1}^{K}\bar{\delta}_i^k \ln[\pi_i(\bar{s}^k, \bar{x}^k : \gamma)] \tag{30}$$

where δ^k, $\bar{s}^k$, and $\bar{x}^k$ are all determined through inspection and γ = parameter to be estimated (Tobin 1958). Estimation of parameter γ, given an amount of $\gamma = (\hat{\gamma}_{10}, \ldots, \hat{\gamma}_{I-1N})$, can be obtained by solving the optimality conditions

$$\frac{\partial \ln[L(\gamma)]}{\partial \gamma_{in}} = 0, \quad (i = 1, \cdots, I-1; n = 0, 1, \ldots, N) \tag{31}$$

that result from maximizing the log-likelihood function (30). The optimal values $\hat{\alpha}_i = \hat{\gamma}_{i0}$ and $\beta_i = (\hat{\gamma}_{i1}, \ldots, \hat{\gamma}_{iN})$ are then estimated by applying a numerical iterative procedure such as the Newton method for the $(I-1)\times(N+1)$ order nonlinear simultaneous equations. Moreover, estimator for the asymptotical covariance matrix of the parameters $[\Sigma(\gamma)]$ is given by

$$\Sigma(\gamma) = \left[\frac{\partial^2 \ln\{L(\gamma)\}}{\partial\gamma\,\partial\gamma'}\right]^{-1} \tag{32}$$

The $(I-1)(N+1)\times(I-1)(N+1)$ order inverse matrix of the right-hand side of the above equation, composed of the element $\partial^2 \ln\{L(\gamma)\}/\partial\gamma_{in}\partial\gamma_{i'n'}$ results in the Fisher information matrix (Greene 2000). In the aforementioned calculation process, it might not be necessary directly to estimate the deterioration state probability $\pi_i(s)$ from the log-likelihood function of Eq. (30). The deterioration state probability can be estimated from multiple integration of Eq. (10). Suffice it to say that the accuracy of estimation for γ depends on the accuracy in calculating the multiple integration. Considering this challenge, in this research we employ double integration, suggested by Steven and Raymond (1997), to improve the accuracy of multiple integral calculation.

Estimation Method for Transition Probability When Having Two or More Visual Inspection Data

In general management practice, the database is composed only of data from two inspection times. However, future monitoring activities may be expanded so as to provide the advantage of data for more than two inspection times. Therefore, besides the estimation methodology for two inspection times as earlier discussed, it is necessary to develop a method to take multiinspection times into account.

For sample k, we assume the condition states $h(\bar{s}_A^k)$ and $h(\bar{s}_A^k + \bar{s}_B^k)$ are, respectively, observed at inspection times $\bar{\tau}_A^k$ and $\bar{\tau}_B^k$. $\bar{\tau}_0^k$ is defined as initial time. Thus, two durations of operation according to two inspection times are further defined as $\bar{s}_A^k = \bar{\tau}_A^k - \bar{\tau}_0^k$ and $\bar{s}_B^k = \bar{\tau}_B^k - \bar{\tau}_A^k$. Additionally, a dummy variable $\bar{\Delta}^k = \{\bar{\delta}_{ij}^k(i = 1, \ldots, I-1, j = 1, \ldots, I)\}$ is determined based on the transition pattern observed from inspections

$$\bar{\delta}_{ij}^k = \begin{cases} 1 & h(\bar{s}_A^k) = i, h(\bar{s}_A^k + \bar{s}_B^k) = j \\ 0 & \text{Otherwise} \end{cases} \tag{33}$$

The information of inspection sample k can be rearranged as $\Xi^k = (\bar{\Delta}^k, \bar{s}^k, \bar{x}^k)$. Since the duration $\bar{s}^k = (\bar{s}_A^k, \bar{s}_B^k)$ is observable, the deterioration state probability can be estimated according to Eqs. (17) and (18). Precisely, the transition probability $\pi_{ij}(\bar{s}_A^k, \bar{s}_B^k)$ can be expressed by means of the function of $\pi_{ij}(\bar{s}_A^k, \bar{s}_B^k : \gamma)$, in which the data $(\bar{s}^k, \bar{x}^k)$ are available from visual inspections, thus making unknown parameter γ the only target of estimation. The description of unknown parameter $\gamma = \{\alpha, \beta_i(i = 1, \ldots, I-1)\}$ is similar to that explained earlier in this section.

In a similar approach to Eq. (30), we define the log-likelihood function for transition probability as follows:

$$\ln[L(\gamma)] = \ln\left[\prod_{i=1}^{I-1}\prod_{j=i}^{I}\prod_{k=1}^{K}\{\pi_{ij}(\bar{s}^k, \bar{x}^k : \gamma)\}^{\bar{\delta}_{ij}^k}\right] = \sum_{i=1}^{I-1}\sum_{j=i}^{I}\sum_{k=1}^{K}\bar{\delta}_{ij}^k \ln[\pi_{ij}(\bar{s}^k, \bar{x}^k : \gamma)] \tag{34}$$

By applying the maximum likelihood estimation approach, we can obtain the value for unknown parameter γ. We will omit a detailed explanation, since this is similar to a reference mentioned earlier in this section. Nevertheless, it is worth emphasizing that the case when $i = I$ is not embedded in the degree of Eq. (34) since I is the absorbing state.

Table 1. Deterioration Condition State Criterion

Inspection result	Condition state	Physical description
OK	1	There is no damage
B	2	The progress of damage is observed. However, damage level is not severe and repair is not necessary.
A	3	There is damage. Repair is recommended. However, urgent repair is not compulsory.
AA	4	Damage is obvious and repair is compulsorily required

Note: If repair is applied, condition state of facility will become 1. The transition of repair is A→OK and AA→OK.

Table 2. Number of Sample Data

Transition pattern	Number of samples	Average operation duration (years)
OK→OK	2	5.24
OK→B	1,321	8.31
OK→A (no historical repair)	10,238 (6,073)	11.98 (9.72)
OK→AA (repair history is available)	750 (4,915)	15.91 (15.36)
Total	12,311	

Note: Nature transition of deterioration is OK→B→A→AA. However, at time τ_0, condition state is OK. The transition in this table reflects the condition of facilities at inspection times.

Expected Deterioration State Probability

The research methodology for deterioration estimation can be applied to every individual infrastructure component. However, in practice, when the deterioration pattern of a large amount of sampling data are considered, it is more convenient to estimate the expected deterioration state probability rather than to focus on that of individual components.

With regard to the relationship between the hazard rate $\theta_i^k(k=1,\ldots,K)$ of sample k and the characteristic variable x, it is understandable to express the distribution function of characteristic variable as $\Gamma(x)$. Thus, statistically, the expected value of the hazard rate $E[\theta_i]$ can be defined by means of the distribution function of characteristic variable x

$$E[\theta_i]=\int_\Theta x\beta_i' d\Gamma(x) \tag{35}$$

where Θ refers to the entire sample population. After averaging the value of the hazard rate, we can again define the Weibull hazard function as

$$\bar{\lambda}_i(y_i)=E[\theta_i]\alpha_i y_i^{\alpha_i-1} \tag{36}$$

Eventually, after the expected hazard rate is estimated from Eq. (36), the expected deterioration state probability [Eqs. (5), (7), (10), and (11)] and the life expectancy of condition states [Eqs. (22) and (26)] can be obtained.

Empirical Analysis

Outline

In this section, we present an empirical application using a tailor program to further verify the applicability of the model, using visual inspection data on the highway tunnel lighting system. Visual inspection was conducted to record the condition of steel board and stainless steel plate (SUS), the two main materials used in the lighting system as reflectors. However, due to the lack of sufficient data on SUS, only results from the visual observation of steel board are used as an application experience. Data concerning the structural visual inspection of highway tunnel lighting system were collected between April 2002 and January 2003. The database also contains information from the opening date. The condition states are ranked by a rating of OK, B, A, and AA, explained in detail in Table 1.

In total, we used 12,311 sample data from the database for empirical analysis. From among the sample data, the transition of condition states in regard to visual inspection times are rearranged in Table 2. The average duration of operation with respect to each condition state counted from the starting time of the infrastructure to the inspection times also shown in the table (inspection time was not the same for every lighting facility). The deterioration pattern is reflected by the transition of deterioration condition states being observed at respective visual inspection times. If the deterioration progress of a lighting facility advances to condition state A or AA, repair can be carried out. However, in the case of condition state A, an immediate repair is not compulsorily required according to the management guideline. While, an immediate repair must be carried out for the facility in condition state AA.

The recorded data also shows the classification of data at the time when visual inspection is carried out. For example, in the total amount of 10,238 samples in condition state A at visual inspection time (group of transition pattern from OK→A), there are 6,073 samples in the group of those without historical repair, 4,165 samples having already received repair in the past. Visual inspection also reveals 750 samples reaching condition state AA, which required immediate repair. Consequently, the total numbers of samples receiving repair action became $4,165+750=4,915$ in the end, and the average operation duration of those facilities reached about 15.36 years.

Hazard Model Estimation

As for physical characteristics, at first, four variables are reviewed as potential candidates, including elapsed time $\bar{s}^k$, type of lighting facility (normal lighting and eased lighting), traffic volume and tunnel inclination. The purpose of combining explanatory variables is to maximize the aforementioned log-likelihood function with a significant level of t-values. Finally, we selected elapsed time and type of lighting as explanatory variables. In addition, we defined the Weibull hazard function as a function of variables as follows:

$$\lambda_i^k(y_i^k)=\alpha_i(\beta_{i0}+\beta_{i1}d^k)(y_i^k)^{\alpha_i-1}(i=1,2,3) \tag{37}$$

In Eq. (37), a dummy variable d^k is added. Its value is defined based on the type of lighting facility. For example, $d^k=0$ is for the case when sample k is a normal lighting facility; otherwise, $d^k=1$. Variable y_i^k indicates the elapsed time over which sample k stays in condition state i. It is noted that variable y_i^k cannot be observed directly. Thus, when we detect $\bar{i}$ as the condition state of sample k, we define the summation of duration as $\Sigma_{m=1}^{\bar{i}} y_m^k=\bar{s}^k$.

Estimation results are presented in Table 3. It can be seen from the table that there is a significant difference between types of lighting facilities. The values of the unknown parameter and its

Table 3. Result of Hazard Model Estimation

Condition state	Multistage Weibull hazard model				Multistage Markovian hazard model			
	α_i	β_{i0}	β_{i1}	$E[\theta_i]$	α_i	β_{i0}	β_{i1}	$E[\theta_i]$
1 (*t* value)	2.039	0.548	−0.323	0.367	1.0	1.054	−0.370	0.847
	(477.54)	(6.14)	(−3.49)	—	—	(10.12)	(−3.66)	—
2 (*t* value)	1.623	0.0812	—	0.0812	1.0	0.265	—	0.265
	(469.92)	(32.90)	—	—	—	(58.99)	—	—
3 (*t* value)	5.709	0.000011	—	0.000011	1.0	0.0882	—	0.0882
	(1,486.69)	(15.10)	—	—	—	(35.43)	—	—
Initial log-likelihood	−811,804.79				−811,804.79			
Log-likelihood	−7,041.67				−8,996.89			
Likelihood ratio	0.991				0.989			

statistical *t-value* associated with the type of lighting facility receives its negative value for Condition State 1. After verification, we recognized the fact that eased lighting, which is located at the tunnel opening, has an early deterioration speed. Thus, the estimation results corresponded exactly to the observed information. Regarding the deterioration of Condition States 2 and 3, estimation results proved that type of lighting facility does not have a significant impact.

Table 3 further displays comparative results between the multistage Weibull hazard model and the multistate Markovian hazard model. The reason behind the comparison is that the multistate Markovian hazard model is in fact a special case of the multistage Weibull hazard model, as when acceleration parameter α in the Weibull hazard function equals 1. It is realized from the table that the acceleration in value of α exactly corresponds to the growth of condition states ($\alpha_1=2.039$, $\alpha_2=1.623$, and $\alpha_3=5.709$). In addition, it is concurrently found that the increase in the elapsed time is in correlation with the increase in value α.

Fig. 5 displays the relationship between elapsed time y_1 of Condition State 1 and the survival probability $\tilde{F}_1(y_1)$ for both normal lighting reflectors ($d^k=1$) and eased lighting reflectors ($d^k=0$). Normal lighting reflectors are referred to lighting reflectors installed in the inside part of tunnels. Eased lighting reflectors are installed in the open part of tunnels (near the gates). It can be seen from the figure that normal lighting reflectors have higher probability of surviving than eased lighting reflectors. The life expectancy of Condition State 1 for eased lighting reflectors is relatively short. For instance, after approximately $y_1 \simeq 1.7$ years in operation, 80% of the total number of eased lighting reflectors in Condition State 1 will change into Condition State 2. On the other hand, 50% of the total number of normal lighting reflectors still remain in Condition State 1.

Fig. 6 shows the distribution pattern of condition states in relation to the duration of operation time of a normal lighting reflector. It is noted that after approximately 6 years in operation, Condition State 1 will be on the verge of disappearing. Based on this finding, it is advisable to implement visual inspection after about 6 years. Moreover, as noted from Table 2, Condition States 3 and 4 account for a large proportion of the sampling population after about 15 years of operation. Therefore, in terms of management, it might be too risky for inspection time to be allocated around the time when there is a high possibility of the onset of Condition States 3 and 4.

Calculation of Management Indicator

Table 4 presents the comparative estimation results for management indicators RMD(i) and RL(i), which are estimated by using multistate Weibull hazard model and multistage Markovian hazard model. It is certain that the values of RMD(i) and RL(i) for Condition States 1 and 2, estimated by the multistage Markovian hazard model, exert only slight differences from that of multistage Weibull hazard model. However, a significant difference between the values of RMD(i) and RL(i) is realized for Condition State 3 when employing the multistage Weibull hazard model [RMD(3) =7.30 and RL(3)=12.95]. This result further proves the impact of elapsed time on estimation results.

A comparison of the values of RL(3) between the two models shows that the value estimated with the multistage Weibull hazard

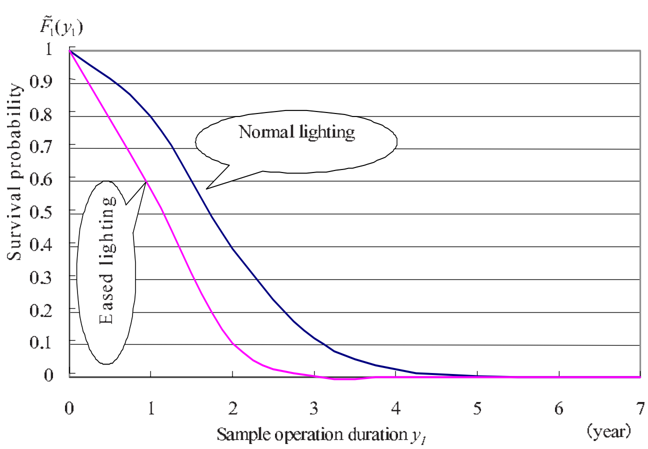

Fig. 5. Survival probability $\tilde{F}_1(y_1)$

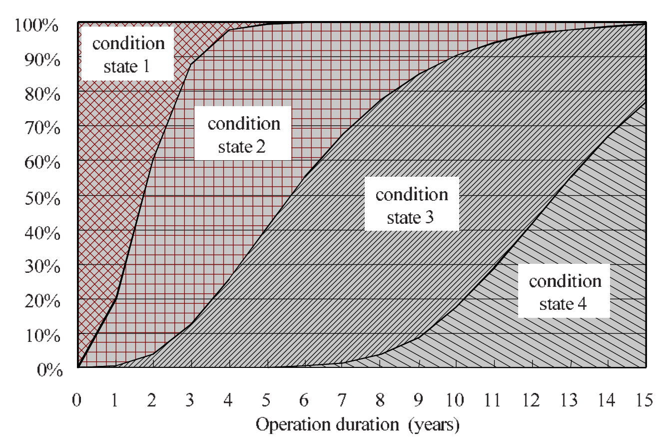

Fig. 6. Deterioration state probability $\pi_i(s)$

Table 4. Management Indicator

Condition state	Life expectancy RMD(i) (years)		Initial life expectancy RL(i) (years)	
	Weibull	Markov	Weibull	Markov
1	1.45	1.23	1.45	1.27
2	4.20	3.77	5.65	5.00
3	7.30	11.34	12.95	16.34

Note: Acronyms for Weibull and Markov in the heading of the table are referred to the multistage Weibull hazard model and multistage Markovian hazard model.

model is shorter than that estimated by using the multistage Markovian hazard model $[\text{RL}(3)=16.34]$. In addition, the average duration measured in Table 2 (15.36) is shorter than that of the multistage Markovian hazard model. These differences are due to the fact that the past operation duration of 4,165 samples with repair in the history were not considered in the multistage Markovian hazard model.

Note that slopes of survival probabilities $\tilde{F}_1(y_1)$ for Condition State 1 along operation duration y_1 drawn for normal lighting and eased lighting reflectors. In addition, note that the relation between operation duration s from initial time and deterioration state probability $\pi_i(s)$ for normal lighting reflectors. Finally, the estimation results for management indicator $\text{RL}_3(h(s_A)=i)$ are shown in Table 5, where values of $\text{RL}_3(h(s_A)=i)$ are presented corresponding to the elapsed time s_A and condition state i. It is noted that the life expectancies shown in the table are only for values of s_A at which the survival probability exceeds 10%. The values presented in the table highlight the fact that when elapsed time s_A increases, the life expectancy of condition states tends to decrease.

Conclusions

This paper has presented a new analytical methodology using the multistage Weibull hazard model for forecasting the deterioration process of infrastructure facilities. The deterioration process is represented by a transition pattern among multiple condition states. In the estimation approach, the maximum likelihood method is employed to estimate the parameters of the model based on observed condition states, characteristic variables and elapsed time of disaggregate samples collected through inspections. The proposed model makes it possible to estimate the transition probability of condition states for any arbitrary time intervals, which was lacked in the past research.

In order to verify the applicability of the model, an empirical study was conducted on a database of tunnel lighting reflectors of express highways in Japan. This study has made a contribution to the field by benchmarking the findings with estimation results using the multistage Markovian hazard model. Estimation results can be used as recommendations for tunnel administrators to work out an optimal inspection plan. For example, it is recommended to implement regular inspection after 6 years since the starting date of lighting reflectors. The analytical methodology presented can be extended to apply not only to tunnel lighting reflectors but to various other kinds of infrastructure facilities as well.

However, we have not discussed several points, which will be considered as topics for extending this study in the future

- Measurement errors occurring in monitoring and inspection activities have not been addressed in this model. In order to tackle this problem, for example, a methodology using Bayesian estimation and Markov Chain Monte Carlo can be incorporated into the model in the future.
- The samples used in empirical study shared almost similar structural characteristics. However, in general practice, an infrastructure database system is often comprised of heterogeneous groups. Thus, the impacts of individual groups on the overall deterioration process should be investigated. A methodology using the mixture mechanism in hazard analysis can be proposed for future consideration.
- In future management, a tendency might develop whereby shorter inspections will become common due to innovations in technology. Hence, the database system of infrastructure management should be designed in such a way that it can be synchronized with an analytical frame. As a sequel, the future focus on multischemes inspection data should be considered.

Table 5. Life Expectancy and Corresponding Condition State $\text{RL}_3(h(s_A)=i)$

Condition state i	$i=1$ (years)	$i=2$ (years)	$i=3$ (years)
$s_A=2$	11.85	10.54	6.40
$s_A=4$	—	9.94	5.77
$s_A=6$	—	9.39	4.92
$s_A=8$	—	9.02	3.96
$s_A=10$	—	—	3.15
$s_A=12$	—	—	2.60

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