Local Mixture Hazard Model: A Semi-Parametric Approach to Risk Management in Pavement System

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***Abstract*— One of the core techniques for the Pavement Management Systems (PMS) is the deterioration hazard model. The hazard models stochastically forecast the deterioration progresses of pavement based on its historical performance indicators over time. This paper introduces a class of hazard models, which consider the heterogeneity factors by a local mixing mechanism. Special attention is paid to the formulation of transition probability of condition state, which is subjected to follow Markov chain. The paper further introduces optimization approach for selecting best repair strategy based on discounted life cycle cost analysis. The application of the model targets to minimize exposing risks to road administrators. The usefulness and practicability of the model are examined through life cycle costs evaluation with real data of Vietnamese highway system.**

***Keywords*— Markov chain, local mixture hazard model**

I. Introduction

Technologies in pavement design and maintenance have been recognized to play a very important role within the framework of the PMS. In many cases, road administrators have to answer the question of selecting for the appropriate technologies and management strategies to suit local requirements, conditions and especially in the existence of budget constrain. Reducing the risks exerted in the process of evaluation and selection, as a matter of course in the PMS, heavily relies on the deterioration forecasting model and the life cycle cost (LCC) analysis since they are widely referred as the heart of PMS.

In regard to the deterioration forecasting model, in recent years, the use of Markov chain based analysis has been one of the major innovations. Hazard model helps users to predict hazard rates, life spans and deterioration curves of infrastructure given the historical inspected condition states and other variables concerning various environment impacts such as: traffic volume, weather, temperature, axle load etc. The application of the Markov chain model has gained its high recognition for its flexibility of modeling and high operability [1].

Deterioration hazard models have largely involved with the application of the Markov chain models. In the Markov chain model, the condition states of infrastructure component are ranked in some discrete states. The values of condition states are recorded through actual inspection over periods of time. And thus, a simulation of deterioration is understood to follow the transit of condition states along with duration.

Further to the application of Markov chain model in PMS, other prominent studies have addressed the application of some statistic distribution such as: Weibull, Gamma, Poison and Exponential distribution families [1], [2]. In addition, in effort to distinguish the hazard rates of different group of similar infrastructure component, heterogeneity factor is embedded in the Markov chain model that significantly provides a good benchmarking method in comprehending the behaviors of a group system. This type of models is named as "Mixture hazard model". However, some literatures have mentioned its limitations and difficulties in estimating heterogeneity factors using probabilistic approach [3]. Thus, a semi-probabilistic approach (local mixture) is presented in this paper aims to address heterogeneity estimation in such circumstance.

Markov decision process is used in the life cycle cost evaluation methods as an excellent approach to propose the best possible choices in the selection of technologies and future maintenance strategies for infastructure [4]. The expected return (cost or duration of time) can be estimated in the integration with Markov chain models and regression estimation. In the PMS, managers often choose the LCC as a method when having to select technological alternative among several proposals. In addition, as a matter of course, the analysis of LCC must take into account of discount factor and management scheme [5].

In this paper, we present a Markov deterioration hazard model used in predicting the deterioration process of road components and the proposal of local mixture hazard model for estimating the heterogeneity factor that exists in different groups of pavement. Furthermore, we discuss the mathematical formulation of repair mechanism and discount LCC analysis based on Markov chain models. An empirical analysis on the Vietnamese highway system is carried out to prove the usefulness and practicability of local mixture hazard model and life cycle cost evaluation methods.

II. MARKOV DETERIORATION HAZARD MODEL

The formulation of deterioration process uses time-homogeneous Markov chain, which is defined on state space $S=\{1,...,I\}$. Here, the rating $i(1,...,I)$ reflexs the healthy status of road sections with $i=1$ to be at its healthiest and $i=I$ to be its worst. The probability, at which the condition state observed at time t expresses as $w(t)=i$ change to $w(t+1)=j$, can be described as:

$$Prob[h(t+1)=j \mid h(t=i]=\pi_{ij} \tag{1}$$

The transition probabilities are expressed by a matrix of dimension *(i,j)*. Here, it is understood that different time intervals give different transition probabilities. The highest level when $i=I$ is called absorbing state. Using the database of past inspections, the transition probability can be estimated [1]. In this section, we only give an outline of the estimation method for ease to readers.

The life expectancy of a condition state i is assumed to be a stochastic variable with the probability density function $f_i(\zeta_i)$ and the distribution function $F_i(\zeta_i)$. The conditional probability that the condition state i at time y_i of a component reaches to condition state $i+1$ at $y_i+\Delta_i$ can be expressed as the hazard function $\lambda_i(y_i)\Delta y_i$.

$$\lambda_i(y_i)\Delta y_i=\frac{f_i(y_i)\Delta y_i}{\tilde{F}_i(y_i)} \tag{2}$$

where $\tilde{F}_i(y_i)=1-F_i(y_i)$ is referred as the survival function of a transition in the condition state i during the time interval $y_i=0$ to y_i. It is assumed that the deterioration process of a component satisfied the Markov property and the hazard function is independent of the time instance y_i on the sample time-axis. Thus, for a fixed value of $\lambda_i>0$, it can be defined.

$$\lambda_i(y_i)=\lambda_i \tag{3}$$

The value of survival function $\tilde{F}_i(y_i)$ can be obtained by using exponential hazard function.

$$\tilde{F}_i(y_i)=\exp(-\lambda_i y_i) \tag{4}$$

Considering the possible inspection time, y_A for instance, the condition state are observed as i at time y_A and keep remaining constant at time $y_A+z_i,(z\geq 0)$. The conditional probability for this event to happen can be defined as

$$\begin{aligned}\tilde{F}_i(y_A+z \mid \zeta_i\geq y_A)&=Prob\{\zeta_i\geq y_A+z \mid \zeta_i\geq y_A\}\\&=\frac{\exp\{-\lambda_i(y_A+z)\}}{\exp(-\lambda_i y_A)}=\exp(-\lambda_i z)\end{aligned} \tag{5}$$

$$\pi_{ii}(z)=Prob[h(\tau_B)=i \mid h(\tau_A)=i]=\exp(-\lambda_i z) \tag{6}$$

where, z indicates the interval between the two inspection times. Equation (6) expresses the probability $Prob[h(y_B)=i \mid h(y_A)=i]$ as the Markov transition probability π_{ii}. Equation (6) also illustrates the hazard rate λ_i and the time interval z are the only two parameter that supports for the calculation of the transition probability π_{ii}.

By defining the subsequent conditional probability of the condition state *i* to *j*, with respect to the actual interval time *z* of inspection, general form for transition probability is as follow.

$$\begin{aligned}\pi_{ij}(z)&=Prob[h(y_B)=j \mid h(y_A)=i]\\&=\sum_{l=i}^{j}\prod_{m=i}^{l-1}\frac{\lambda_m}{\lambda_m-\lambda_l}\prod_{m=l}^{j-1}\frac{\lambda_m}{\lambda_{m+1}-\lambda_l}\exp(-\lambda_l z)\\&(i=1,\cdots,I-1;\, j=i+1,\cdots,I)\end{aligned} \tag{7}$$

For convenience of mathematical manipulation, we define

$$\prod_{m=i,\neq l}^{j-1}\frac{\lambda_m}{\lambda_m-\lambda_l}\exp(-\lambda_l z)=\prod_{m=i}^{l-1}\frac{\lambda_m}{\lambda_m-\lambda_l}\prod_{m=l}^{j-1}\frac{\lambda_m}{\lambda_{m+1}-\lambda_l}\exp(-\lambda_l z)$$

The life expectancy of rating $RMD_i(x)\,(i=1,\cdots,I-1)$ and the everage life expectancy ET_j $(j=2,\cdots,I)$ at elapsed time from i=1 and advances to state j(>1) can be expressed as

$$RMD_i(x)=\int_0^{\infty}\tilde{F}_i(y_i)dy_i=\int_0^{\infty}\exp\{-\lambda_i y_i\}dy_i=\frac{1}{\lambda_i} \tag{8}$$

$$ET_j=\sum_{i=1}^{j}\frac{1}{\lambda_i} \tag{9}$$

This is important indicator to draw the deterioration curves of investigate infrastructure components

III. LOCAL MIXTURE HAZARD MODEL

A. Mixing Distribution of Hazard Rate

It has been realized that similar group of individual infrastructure components can be exerted to have different deterioration speeds. To express these differences, the term "heterogeneity factor" is employed. Here, the letter ε^k denotes the heterogeneity parameter, which infers the change of characteristic of a peculiar hazard rate to a pavement component $k\,(k=1,\cdots,K)$. Thus, the mixture index hazard function (3) can be expressed as

$$\lambda_i^k=\tilde{\lambda}_i^k\varepsilon^k\ (i=1,\cdots,I-1;k=1,\cdots,K) \tag{10}$$

Here, the value of ε^k is always greater than 0. Importantly, the deterioration speed of component k is fast when value of ε^k increased in comparison with the rate of standard hazard $\tilde{\lambda}_i^k$. It is also noted from (10) that the same random variable ε^k is included in the mixture index hazard function of all ratings. ε^k is understood to be in a form of function or stochastically distribution.

Here, it is necessary to set value of $\varepsilon^k (k = 1, \cdots, K)$ to $\bar{\varepsilon}^k$. Hence, the probability (4) that the life expectancy of the rating i of component k becomes y_i^k or more can be rewritten by using the index hazard function (10) as

$$\tilde{F}_i(y_i^k) = \exp(-\tilde{\lambda}_i \bar{\varepsilon}^k y_i^k) \quad (11)$$

In addition, the Markov transition probability that the condition state i at time y_A^k of component k remains unchanged at time $y_B^k = y_A^k + z^k$ will be

$$\pi_{ii}^k(z^k : \bar{\varepsilon}^k) = \exp(-\tilde{\lambda}_i^k \bar{\varepsilon}^k z^k) \quad (12)$$

Moreover, the Markov transition probability $\pi_{ij}^k(z^k : \bar{\varepsilon}^k)$ that the rating changes to $j (> i)$ between y_A^k and $y_B^k = y_A^k + z^k$ will be formed from (7) and (10) as

$$\begin{aligned} \pi_{ij}^k(z^k : \bar{\varepsilon}^k) &= \sum_{l=i}^{j} \prod_{m=i, \neq l}^{j-1} \frac{\tilde{\lambda}_m^k}{\tilde{\lambda}_m^k - \tilde{\lambda}_l^k} \exp(-\tilde{\lambda}_l^k \varepsilon^k z^k) \\ &= \sum_{l=i}^{j} \psi_{ij}^l(\lambda^k) \exp(-\tilde{\lambda}_l^k \varepsilon^k z^k) \qquad k = 1, \cdots, K \end{aligned} \quad (13)$$

where,

$$\psi_{ij}^l(\lambda^k) = \prod_{m=i, \neq l}^{j-1} \frac{\tilde{\lambda}_m^k}{\tilde{\lambda}_m^k - \tilde{\lambda}_l^k} \quad (14)$$

Equation (14) expresses the dependence only on the function of average hazard rate.

B. Local Mixture Hazard Model

Regarding the heterogeneity factor ε^k, it is not easy to infer ε^k to follow a particular function. Several investigations were on assuming function of ε^k as stochastic distribution such as: Gamma, Lognormal distributions etc. However, the results are not always satisfied in fitting with actual transition. In order to cope with this obstacle, one possible rule is to consider the form taken by mixture distribution as when ε^k has little dispersion so that the departure from homogeneity is small [3]. One of the definitions of the local mixture model is as follows:

The local mixture model of a regular exponential family $f(x;\varepsilon)$ is defined via its mean parameterisation as

$$g(x;\varepsilon) := f(x;\varepsilon) + \sum_{i=2}^{r} f^k(x;\varepsilon) \quad (15)$$

where $$f^k(x;\varepsilon) = \frac{\delta^k}{\delta \varepsilon^k} f(x;\varepsilon)$$

Another class of local mixture model that captures the behavior of scale dispertion mixture of $f(x;\varepsilon)$ particularly when the dispersion parameter ε is small, is defined as local scale mixture model in the following equation

$$g(x;\varepsilon) := f(x;\varepsilon) + \sum_{i=2}^{r} \frac{\varepsilon^k}{k!} f^k(x;\varepsilon) \quad (16)$$

It is noted in equations (15) and (16) that, the parameterisation is understood to be follow Taylor series. Further, from (12) and (13), the transition probability (or the density distribution function) has following form

$$\tilde{\pi}_{ij}(z) = \int_0^\infty \pi_{ij}(z : \varepsilon) f(\varepsilon) d\varepsilon \quad (i = 1, \cdots, I-1) \quad (17)$$

For ease of mathematical expression, let assume the local mixture transition probability as the exponential function $f_m(\varepsilon, z, \lambda)$ with m as indication for local mixture. Equation (17) can be simplified as

$$f_m(\varepsilon, z, \lambda) = \int f(\varepsilon, z, \lambda) dH(\varepsilon) \quad (18)$$

where, *dH(ε)* is arbitrary distribution around the mean of 1. $f(\varepsilon, z, \lambda)$ is exponential family, and thus, can be further expressed as *f(ε,z,λ)=exp(-ελz)*. The expected value of *f(ε,z,λ)* with respect to ε is likely a function of ε about its mean, which, as noted earlier in equation (16), can be taken to be unity with no loss of generality as long as the mean exits, to get

$$exp(-\varepsilon\lambda z) = e^{-\lambda z}(1 + (\varepsilon - 1)(-\lambda z) + \frac{(\varepsilon - 1)^2}{2!}(-\lambda z)^2 + ... \quad (19)$$

Without lossing generality, the expectation of this series can be expressed in quadratic form (equivalent to *r = 2* in equation (16). This assumption is proved to exponential family as it turn to produce very attractive statistical property. Thus, a quadractic form gives

$$E(e^{-\varepsilon\lambda z}) \approx e^{-\lambda z}\{1 + \frac{\sigma^2(\lambda z)^2}{2}\} \quad (20)$$

From this approximation approach, equation (12), (13) and (17) can be understood in the following forms

$$\tilde{\pi}_{ii}(z) = e^{-\tilde{\lambda}_i z}\{1 + \frac{\sigma^2(\tilde{\lambda}_i z)^2}{2!}\} \quad (21)$$

$$\tilde{\pi}_{ij}(z) = \sum_{l=i}^{j} \psi_{ij}^l(\tilde{\lambda}) e^{-\tilde{\lambda}_l z}\{1 + \frac{\sigma^2(\tilde{\lambda}_l z)^2}{2!}\} \quad (22)$$

C. Presumption of Local Mixture Transition Probability

The deterioration process of sample k can be expressed by using mixture index hazard function $\lambda_i^k(y_i^k) = \tilde{\lambda}_i^k \varepsilon^k$, and at I is absorbing state, where $\pi_{II}^k = 1$, $\tilde{\lambda}_I^k = 0$. The hazard rate $\tilde{\lambda}_i^k$ depends on the characteristic vector of road component and suppose to change to the vector x^k as follows

$$\tilde{\lambda}_i^k = x^k \beta_i' \quad (23$$

where $\beta_i = (\beta_{i,1}, \cdots, \beta_{i,M})$ is a row vector of unknown parameters and the symbol $'$ indicates the vector is transposed. From (21) and (22), the standard hazard rate in each rating can be expressed by the mean of the probability distribution of hazard rate $\tilde{\lambda}_i^k$ and the heterogeneity ε^k. The average Markov transition probability is expressible by (22) when using row vector $\overline{x}^k$ of the infrastructure component ($\overline{x}^k = (\overline{x}_1^k, \cdots, \overline{x}_M^k)$ indicating the observed value of variable *m* for the sample *k*). In addition, the transition probability also depends on the inspection time interval $\overline{z}^k$ when data is observed. Thus, it is $\tilde{\pi}_{ij}^k(\overline{z}^k, \overline{x}^k : \theta)$ with $(\overline{z}^k, \overline{x}^k)$ and $\theta = (\beta_1, \cdots, \beta_{I-1}, \sigma)$ for the average Markov transition probability $\tilde{\pi}_{ij}^k$.

$$L(\theta, \Xi) = \prod_{i=1}^{I-1} \prod_{j=i}^{I} \prod_{k=1}^{K} \left\{ \tilde{\pi}_{ij}^k(\overline{z}^k, \overline{x}^k : \theta) \right\}^{\overline{\delta}_{ij}^k} \tag{23}$$

where, $\delta^k = (\overline{\delta}_{11}^k, \cdots, \overline{\delta}_{I-1,I}^k)$ is a dummy variable vector and takes value *1* at $h(\overline{t}^k) = i, h(\overline{y}^k) = j$ and *0* otherwise. Since $\theta = (\beta, \sigma)$, and $\tilde{\pi}_{ij}^k(\overline{z}^k, \overline{x}^k : \theta)$ is a rating transition probability at the initial time. It can be expressed as

$$\tilde{\pi}_{ii}^k(\overline{z}^k, \overline{x}^k : \theta) = e^{-\overline{x}^k \beta_i' \overline{z}^k} \{1 + \frac{(\sigma \overline{x}^k \beta_i' \overline{z}^k)^2}{2!}\} \tag{24}$$

$$\tilde{\pi}_{ij}(\overline{z}^k, \overline{x}^k : \theta) = \sum_{l=i}^{j} \psi_{ij}^l(\lambda) e^{-\overline{x}^k \beta_l' \overline{z}^k} \{1 + \frac{(\sigma \overline{x}^k \beta_l' \overline{z}^k)^2}{2!}\} \tag{25}$$

$$(i = 1, \cdots, I-1; j = i+1, \cdots, I)$$

where,

$$\psi_{ij}^l(\lambda^k) = \prod_{m=i, \neq l}^{j-1} \frac{\overline{x}^k \beta_m'}{\overline{x}^k \beta_m' - \overline{x}^k \beta_l'} \tag{26}$$

Since $\overline{\delta}_i^k$, $\overline{z}^k$, $\overline{x}^k$ are known from the inspection, the likelihood functions are functions of β, μ. In the method of maximum likelihood, $\hat{\theta} = (\hat{\beta}, \hat{\sigma})$ that maximizes (23) will be presumed. Functions (23) can be defined as the log-likelihood function as follow

$$\ln L(\theta, \Xi) = \sum_{i=1}^{I-1} \sum_{j=1}^{I} \sum_{k=1}^{K} \overline{\delta}_{ij}^k \tilde{\pi}_{ij}^k(\overline{z}^k, \overline{x}^k : \theta) \tag{27}$$

The estimation of $\hat{\theta} = (\hat{\theta}_1, \cdots, \hat{\theta}_{(I-1)M})$ can be obtained by solving the optimality conditions

$$\frac{\partial \ln L(\theta, \Xi)}{\partial \theta_i} = 0, \qquad (i = 1, \cdots, (I-1)M) \tag{28}$$

When the value of $\hat{\theta}$ is obtained, from equation (18), the following likelihood function to estimate value of ε^k, which is given as $\hat{\varepsilon}^k$, is formulated

$$\begin{aligned} \ln \ell^k(\xi^k : \hat{\theta}, \varepsilon^k) &= \ln f(\varepsilon^k : \hat{\sigma}) \\ &+ \sum_{i=1}^{I-1} \sum_{j=i}^{I} \overline{\delta}_{ij}^k \pi_{ij}^k(\overline{z}^k, \overline{x}^k : \hat{\beta}, \varepsilon^k) \quad (k = 1, \cdots, K) \end{aligned} \tag{29}$$

The optimum value of ε^k can be obtained by solving

$$\frac{\partial \ln \ell^k(\xi^k : \hat{\theta}, \varepsilon^k)}{\partial \varepsilon^k} = 0 \tag{30}$$

IV. Best Repair based on Cost optimization Principle

This section briefly explains the formation of the LCC estimation based on certain pre-determine repair actions. The details of calculation can be referred to Otazawa 2005 [6].

The expectation cost denoted as ψ(i) is expressed by the period concerning the value at time *(t+1)*. And *ψ(i) (i=1,...,K-1)* represents a table of aspect as for the minimum value of expectation cost (LCC) generated when the best repair action d^* is executed after time *(t+1)*. The expectation cost $\Omega^d(i)$ generated at time *t* and deteriorated condition state is *j* will be calculated from (31).

$$\Omega^d(i) = e_i^d + \delta E_i^d[\Psi(j)] \tag{31}$$

Here, when the condition state at time *t* is *i*, e_i^d shows the expectation repairing cost required under the repair strategy *d* just before time *(t+1)*.

$$e_i^d = \sum_{j=i+1}^{K} p_{ij} c_j^d \;\; (i = 1, \cdots, K-1) \tag{32}$$

In equation (31), $\delta \, (0 < \delta < 1)$ is discount factor and $E_i^d[\Psi(j)]$ is cost of the expectation life cycle generated under repair strategy *d* after time $(t+1)$ (evaluated by period concerning the value at time *t+1)* in time *t* when deteriorated condition state is *i*.

$$E_i^d[\Psi(j)] = \sum_{j=1}^{K-1} p_{ij}^d \Psi(j) \tag{33}$$

The superscript notation expreses the process under the repair rule *d*. Expected cost in the life cycle evaluation is often estimated at a particular time *t* (*t* can be at the present or just at the time after the first repairing action). Thus, in LCC analysis, recurrently defining the minimum value of expectation cost at time *t* can result in the optimal solution. The regression equation to determine the optimal expectation cost is as follow

$$\Psi(i) = \min_{d \in D} \delta \{ e_i^d + E_i^d[\Psi(j)] \} \qquad (i, j = 1, \cdots, K-1) \tag{34}$$

It is from (34) to note that the best management (or repair) strategy d^* can be defined as an action vector η^{d*}. Of which, the (i) element is in best repair action $\eta^{d^*(i)}$ satisfying (34) in *(k-1)* condition states. The optimality can be solved by applying dynamic programming.

V. Empirical Analysis

A. Outline

In this section, we present empirical analysis of local mixture model and maintenance optimization used cost minimization principle discussed in previous sections. We use two set of visual inspection data on the Vietnamese national highway system in the year 2001 and 2004. After validating the inspected data, we choose 1237 highway sections as candidate samples. We further categorized 1237 sections into 8 groups according to their similarity in term of the traffic volume and the asphalt overlay thickness. The range of rating is also defined for the Markov model since there has been no national standard of rating for highway in Vietnam. We selected a range of rating in the space of *S=(1,...,5)*. Table I and Table II present the method to convert corresponding the International Roughness Index (IRI) into the range of 5 ratings. In which, rating *i=1* is to be the best (the healthiest) condition of the highway section. And *i=5* to be the worst situation.

TABLE I. CONDITION STATE AND IRI (TCN-N22 2001 [7])

Design Speed	IRI (m/km)			
(Km/h)	*Good*	*Fair*	*Poor*	*Very Poor*
120; 100; 80	(0-2)	(2-4)	(4-6)	(6-8)
60	(0-3]	(3-5]	(5-7]	(7-9)
40; 20	(0-4]	(4-6]	(6-8]	(8-10)

TABLE II. NOTATIONS OF NEW CONDITION STATE

Condition State	Equivalent IRI	Remark
1	(1-2)	Very good
2	(2-4)	Good
3	(4-6)	Fair
4	(6-8)	Poor
5	> 8	Very Poor

B. Transition Probabilities and Deterioration Curve

In the first step, we estimated the average transition probabilities of the highway system by using hazard model that is explained in (7). The two parameter used for estimation are the thickness of overlay asphalt (x_2) and the traffic volume (x_3), which are subjected to change over two time point of inspection. In general, the general form of the hazard function is explainable by $\lambda_i^k = \beta_{i,1} + \beta_{i,2}x_2^k + \beta_{i,3}x_3^k$ with *(i=1,...,4; n=1,...,N)*, with β, *N* indicating unknown parameters and the number of samples respectively. Table III shows the result of the maximum likelihood estimations, in which, the value of unknow parameters $\hat{\beta}$ are obtained with the respective *t*-values of each explanatory variable.

TABLE III. EXPONENTIAL HAZARD MODEL RESULTS

State	Absolute $\beta_{i,1}$	Surface Thickness $\beta_{i,2}$	Traffic Volume $\beta_{i,3}$
1	0.3052 (26.027)	-	-
2	0.1792 (2.6557)	0.4996 (2.4433)	1.0570 (2.4174)
3	-	2.6461 (9.6294)	-
4	-	1.8089 (6.5421)	-

Note) *t*-values are shown in parenthesis

TABLE IV. AVERAGE TRANSITION PROBABILITIES MATRIX

Condition State	1	2	3	4	5
1	0.5431	0.2773	0.0998	0.0574	0.0224
2	0	0.3756	0.2522	0.2348	0.1374
3	0	0	0.1673	0.4022	0.4305
4	0	0	0	0.2945	0.7055
5	0	0	0	0	1

Table IV shows the average transition probability when taking the whole set of data into calculation of the exponential hazard model. The values are coressponding to the mean of 1 for the heteroneity factor. Benchmark deterioration curve is drawn as a result of this transition probability.

In the second step, the local mixture model is applied to estimate the heterogeneity factor ε^k. In this application, the number of groups is 8. The values of ε^k are estimated to be (0.6495, 0.9909, 0.9957, 0.5109, 1.1931, 0.5665, 0.5112, 1.1545) respectively to particular group. Thus, a combination of step 1 and step 2 makes it possible to draw the deterioration curves respectively in "Fig. 1".

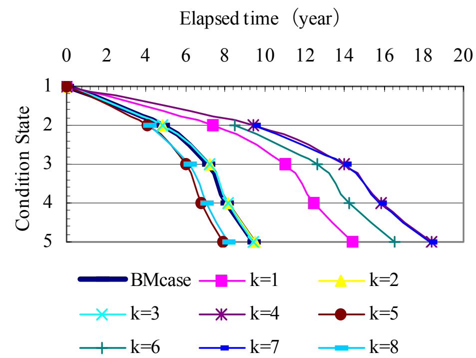

Figure 1. Expected deterioration path for each group

Result in "Fig. 1" enables us to have a good comparison of the dispersion of each group of highway with respect to the thickness of surface asphal and the traffic volume around the average deterioration curve. The life expectancy of rating and deterioration curve are important indicators, which assist managers to select the desire representative highway for the life cycle cost analysis. In our study, the group with heteroneity factor ε = 1.1931 (k=5) is selected for sample application of the LCC analysis. Table V shows the results of transition probability for this group.

TABLE V. TRANSITION PROBABILITIES MATRIX FOR K = 5

Condition State	1	2	3	4	5
1	0.7805	0.1700	0.0328	0.0132	0.0035
2	0	0.5996	0.2136	0.1354	0.0514
3	0	0	0.2767	0.4334	0.2899
4	0	0	0	0.4062	0.5938
5	0	0	0	0	1

C. Maintenance Choices and Optimal Policies

Major maintenance and repair (M&R) is selected when the condition of highway advances into severe deterioration [8]. Major M&R for asphalt highway requires to remove the wear layers and provide a good bonding with overlay. Several methods of construction technique in asphalt materials (cold milling, cold recycling, hot recycling and asphalt concrete overlay) and the choice of thickness makes it possible to propose many M&R alternatives. According to the standard practice in design and construction of asphalt concrete highway [7], the construction norm in maintenance and repairing [9] and construction unit price of Hanoi [10], we come up with a table of cost that relatively reflects the actual repairing cost by applying different thickness of overlay to particular condition state (refer to Table VI).

TABLE VI. REPAIRING COST

$1000 dollar

Repair actions	Condition state				
	1	*2*	*3*	*4*	*5*
Do nothing	0.00	0.00	0.00	0.00	0.00
Routine maintenance (0.5 kg bitumen/m2)	1.13	1.88	0.00	0.00	0.00
Routine maintenance (0.8 kg bitumen/m2)	1.59	2.65	0.00	0.00	0.00
Routine maintenance (1 kg bitumen/m2)	1.89	3.15	0.00	0.00	0.00
Routine maintenance (1.5 kg bitumen/m2)	2.64	4.41	0.00	0.00	0.00
3 cm AC overlay	0.00	0.00	21.32	24.09	29.20
4 cm AC overlay	0.00	0.00	24.39	27.57	33.42
5 cm AC overlay	0.00	0.00	30.61	34.60	41.93
6 cm AC overlay	0.00	0.00	36.98	41.80	50.66
7 cm AC overlay	0.00	0.00	44.47	50.26	60.92

Note: Cost only applies for highway class I, one lane of 7 m in width and 1000 m in length. User-cost is not included

Since the differences in thickness of overlays result in different deterioration rates. Thus, life cycle cost analysis for respective thickness of overlays would enable us to select the best optimal thickness. We only focus on using 5 cm thickness of overlays as major M&R to apply for different condition state. As noted in Table V, when $i=1$, $i=2$, there is not much deterioration. In condition state $i=3$, the deterioration state to be faster. Thus, a localized safety and global preventive maintenance are suggested to apply as most effective. Major M&R will be carried out for condition state advance from $i=3$ to $i=5$. In this understanding, the total number of major M&R is 3. We notates the major M&R by space vector $D=(d1, d2, d3)$, with *d1, d2, d3* as repair actions applied to condition state $i=3$, $i=4$, and $i=5$ respectively. The horizon of one management term is assumed to be in 20 years, the discounted factor δ is equal to 0.8. Finally, result of LCC analysis for repair actions $d1, d2$ and $d3$ is calculated and presented in "Fig. 2". The optimal repair rule is expressed by space $d^* = (d3, d3, d3, d3, d3, d3, d3, d3, d3, d3, d3, d3, d3, d3, d3, d3, d3, d3, d3)$ since this rule gives the minimum expectation cost .

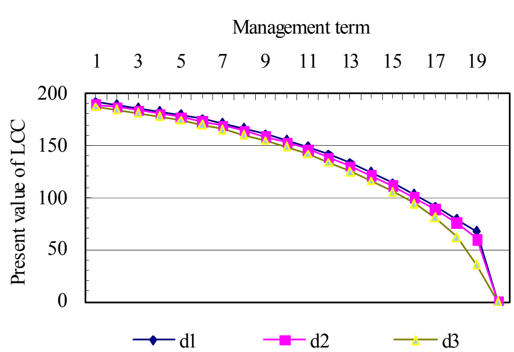

Figure 2. Present value of life cycle cost incurring at respective years under repair rules

VI. CONCLUSION

This study illustrates the mathematical formation of the Markov chain model, which is expressible by using the exponential form of the hazard function. The local mixture hazard model is discussed as a semi-probabilistic approach to estimate the heterogeneity factors that exists in a group of highways. The combined application of the local mixture model and the optimization model assists road administrators in benchmarking study and risk management. Empirical analysis was conducted in order to recommend the best asphalt surface structure. However, due to the limitation of data for the entire road network, we only stop at giving advise of the best possible repair strategy for asphal layer of 5 cm thickness when the management term assumed to end after 20 years. The application of model can be extended for a large-scale of benchmarking study and risk management when having sufficient data.

REFERENCES

[1] Y. Tsuda, K. Kaito, K. Aoki and K. Kobayashi, "Estimating markovian transition probabilities for bridge deterioration forecasting," JSCE Journal (in Japanese), no. 801/I-73, October 2005.

[2] H. C. Shin and S. M. Madanat, "Development of stochastic model of pavement distress initatation," Processing of JSCE Conference, vol. 744, pp. 61-67, 2003.

[3] K. Anaya and P. Marriott, "Local mixtures of the exponential distribution," Annals of the Institute of Statistical Mathematics, vol. 59, no. 1, pp. 111-134, 2007.

[4] D.J. While, Markov Decision Process, Springer, 1992.

[5] K. Kobayashi, "Cost evaluation decentralization of life cycle cost, total efficiency," JSCE Journal (in Japanese), no. 793/IV-68, pp. 59-71, 2005

[6] T. Otazawa, K. Ishihara, K. Kobayashi and Y. Kondo, "The best mending policy in consideration of economical longevity," JSCE (in Japanese), vol. 772/IV-65, pp. 169-184, 2004.

[7] TCN-N22, Vietnamese Standards for Design and Construction of Highway- 22TCN-277-01, Ministry of Transportation, Vietnam, 2001.

[8] M. Y. Shahin, Pavement Management for Airports, Roads, and Parking Lots, Springer, 2005.

[9] DM1242, Construction Norm, Ministry of Construction, Vietnam, 1998.

[10] DG24, Construction Unit Price, Construction Book, Hanoi, 2007