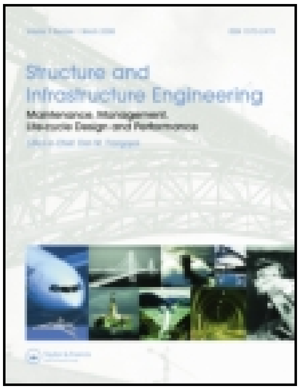

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In this paper, a probabilistic model for the determination of optimal intervention strategies (OISs) for a road link composed of multiple objects that are affected by gradual deterioration processes is investigated. The model is composed of a deterioration part and a strategy evaluation part. In the deterioration part, a Weibull hazard function is used to represent the deterioration of the individual objects, where the values of the model parameters are to be estimated using inspection data. A threshold condition state (CS) for each object is defined, at which an intervention must be executed. The results of the deterioration part are used as inputs in the strategy evaluation part, in which OISs for individual objects and for the link as a whole are determined. The determination of the optimal strategies takes into consideration impacts on multiple stakeholders. The model is demonstrated by determining the OISs for a fictive road link composed of one bridge and two road sections. The main strengths of the methodology are that past deterioration is taken into consideration and that it is possible to consider the execution of interventions simultaneously and, therefore, associated reductions in impacts that normally occur when interventions are grouped. The main weakness of the methodology is that the condition of the objects is represented using only two CSs, i.e. fully operational and not fully operational.

Keywords: Weibull analysis; total cost analysis; multi-stakeholder approach; road asset management

1. Introduction

Inadequate performance of an object, e.g. a bridge or a road section, in a road link that results in the inability of the link to provide an adequate level of service results in negative impacts on the stakeholders of the road. In order to manage the road link in a way to minimise negative impacts on stakeholders, i.e. to determine optimal intervention strategies (OISs), it is necessary to understand how the objects in the link will change over time and to determine the optimal times to execute an intervention and the types of interventions to be executed at these times.[2]

The process of determining the impacts on stakeholders that are incurred when intervention strategies (ISs) are followed is often referred to as life cycle cost (LCC) analysis (Kobayashi & Kuhn, 2007; Woodward, 1997). The results of analyses are then used to determine the strategy that results in the lowest overall costs over the life cycle of the object (Adey et al., 2010; Jido, Otazawa, & Kobayashi, 2008; Kobayashi & Kuhn, 2007). One important, but subtle, feature of the process of determining the OIS is that it is dependent on the deterioration models used to predict the deterioration of an object. Deterioration models used for this purpose can be classified as those that (1) estimate the average deterioration of all similarly classified objects or elements and (2) estimate the deterioration of each object or element. For the former, statistical methods are often used, and for the latter, methods based on deterministic physical models are often used.

In this paper, a methodology for the determination of OISs for a road link composed of multiple objects, where the condition of each object is represented using two condition states (CSs), is proposed. The methodology is composed of a deterioration part and a strategy evaluation part. In the deterioration part, a Weibull hazard function is used in a probabilistic model to represent the deterioration of the individual objects, where the values of the model parameters are to be estimated using inspection data. A probabilistic deterioration model was selected to take into consideration the uncertainty in deterioration prediction, which is increasingly seen as both possible and necessary in order to make appropriate decisions on when interventions should be executed (Frangopol, Kallen, & Noortwijk, 2004; Frangopol & Neves, 2008; Jido et al., 2008; Madanat, Bulusu, & Mahmoud, 1995; Nakat & Madanat, 2008). It is seen as increasingly possible due to the increasing ease with which sufficient data can be collected, stored and processed in order to determine

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correct probabilistic models (Ker, Lee, & Wu, 2008; Wang, Mahboub, & Hancher, 2005). It is seen as increasingly necessary because the correct determination of OISs can result in significant reductions in the negative impacts related to road use and it is not possible to predict the deterioration of an object exactly, i.e. deterministically. Coupled with this increasing necessity is also an increasingly wide spread use of probabilistic models in infrastructure management decision making (Frangopol et al., 2004; Frangopol & Neves, 2008), which has the additional benefit of increasing their acceptance in management decision making. For example, it is common that state-of-the-art bridge management systems such as KUBA and PONTIS use probabilistic models (FHWA, 2005). The models used in these systems are often based on Markov chain theory, meaning that a range of discrete CSs are used to represent the physical condition of civil infrastructure objects; it is assumed that the transition probabilities are stationary and the historical performance of the object is not directly considered.

The results of deterioration part are used as inputs in the strategy evaluation part, in which OISs for individual objects and for the link as a whole are determined. The determination of the OIS takes into consideration impacts on multiple stakeholders. To demonstrate the functioning of the methodology, the OIS for a road link composed of three objects is determined.

2. Methodology

2.1. Deterioration part

To model deterioration, it is assumed that an object can be in only one of the two CSs: (1) fully operational or (2) not fully operational; that the transition of the object between states can be described by a stochastic variable $\tau \in [0, \infty]$ that represents the time to depart from CS1 (Lancaster, 1990) and that the probability of transition of an object from state 1 to state 2 can be represented by a Weibull distribution function. The Weibull distribution function is used because it is not memoryless, overcoming some of the criticisms of the widely used exponential distribution (Gertsbakh, 1997; Gertsbakh, 2000; Marquez, 2007), and since it has been found to be a good representation of certain deterioration processes in the past (Agrawal, Kawaguchi, & Chen, 2010; Kobayashi & Kaito, 2010; Kobayashi, Kaito, & Lethanh, 2010). It is noted that the suitability of this model should be checked using the data related to the specific objects in question.

The deterioration and intervention (or renewal) process is illustrated in Figure 1. Time t_k is the duration that the object is in CS1. When the object reaches CS2, intervention is required to bring it back to CS1, and the cycle of deterioration and intervention process is repeated. As τ is a stochastic variable, it has a probability distribution function $F(\tau)$ and a probability density distribution $f(\tau)$. The probability of remaining in CS1 (hereafter referred to as survival probability) expressed by survival function $\tilde{F}(\tau)$ can be defined according to the value of failure probability $F(\tau)$ as:

$$\tilde{F}(\tau) = 1 - F(\tau). \tag{1}$$

The probability of the object being in CS1 until time τ and then entering CS2 for the first time during the interval $\tau + \Delta\tau$ can be regarded as the hazard function, which is given by:

$$\lambda_i(\tau)\Delta\tau = \frac{f(\tau)\Delta\tau}{\tilde{F}(\tau)}, \tag{2}$$

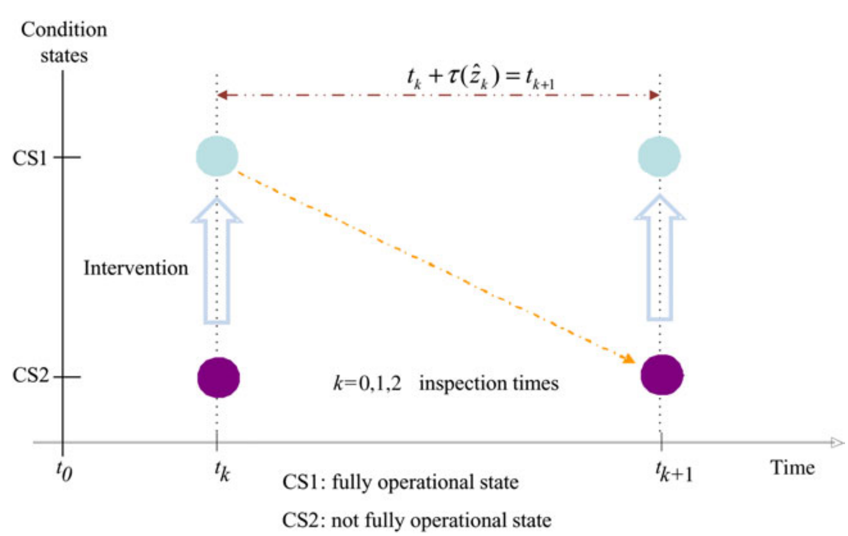

Figure 1. Deterioration and intervention process.

the probability that the object transitions from CS1 to CS2 depend greatly on the elapsed time that it has spent in CS1, i.e. the hazard function has a memory. Using the Weibull distribution function, the hazard function is given by:

$$\lambda(\tau) = \alpha m \tau^{m-1}, \tag{3}$$

where α is the so-called arrival density and m is the acceleration or shape parameter (Lancaster, 1990).

The parameter α can be expressed in multiplicative form of the unknown parameter β and the characteristic variables (covariates) x, i.e. the factors that affect the rate of deterioration, for example the daily traffic volume (DTV), ambient temperature and pavement thickness.

$$\alpha = \sum_{i=1}^{M} \beta_i x_i, \quad (i = 1, \ldots, M), \tag{4}$$

where M is the total number of covariates and the value of first covariate is 1[3].

The probability density function $f(\tau)$ and survival function $\tilde{F}(\tau)$ of the Weibull hazard function are given by Equations (5) and (6), respectively:

$$f(\tau) = \alpha m \tau^{m-1} \exp(-\alpha\tau^m), \tag{5}$$

$$\tilde{F}(\tau) = \exp(-\alpha\tau^m). \tag{6}$$

In order to obtain the values of the parameters α and m, the maximum likelihood estimation method can be used, where the parameter values $(\theta_1 = \alpha, \theta_2 = m)$, which maximise the logarithmic likelihood function (9) $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$, i.e. satisfy:

$$\frac{\partial \ln \mathcal{L}(\Xi, \hat{\theta})}{\partial \theta_i} = 0, \quad (i = 1, 2), \tag{7}$$

where L is the maximum likelihood function, Ξ is the set of observed data and the most likely values of $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ are estimated by using numerical iterative procedures such as Newton method [4] for simultaneous equations (Equation (8)) (Kelley, 1999). In order to test these values for statistical significance, the probabilistic t-test and the asymptotic covariance matrix (Equation (8)) [5] can be used (Cramer, 1946).

$$\hat{\sum}(\hat{\theta}) = \left[\frac{\partial \ln L(\Xi, \theta)}{\partial \theta \partial \theta'}\right]^{-1}. \tag{8}$$

To be clear about the maximum likelihood estimation method, the likelihood of the values of the model parameters, given the set of observed data Ξ, e.g. CSs, time to failure of all similar object, $s(s = 1, \ldots, S)$ and assuming that the deterioration of each object is independent from all other objects, is given by:

$$\ln \mathcal{L}(\alpha, m : t_s) = \sum_{s}^{S} \left[\begin{array}{c} (1-\delta_s)(-\alpha t_s^m) \\ +\delta_s\{\ln\alpha + \ln m + (m-1)\ln t_s - \alpha t_s^m\} \end{array} \right]. \tag{9}$$

where δ_s is a binary variable which has value of 1 when CS2 is observed and 0 otherwise and t_s is used to represent the evolution of time for object s.

For ease of mathematical manipulation, the logarithm of both sides of Equation (9) is taken and often referred to as the log-likelihood function.

$$\begin{aligned}\mathcal{L}(\alpha, m : t_s) &= \prod_{s}^{S} \left\{\bar{F}(t_s^m)\right\}^{(1-\delta_s)} \left\{f(t_s^m)\right\}^{\delta_s} \\ &= \prod_{s}^{S} \left\{\exp(-\alpha t_s^m)\right\}^{(1-\delta_s)} \left\{\alpha m t_s^{m-1} \exp(-\alpha t_s^m)\right\}^{\delta_s}.\end{aligned} \tag{10}$$

2.2. Evaluation part

2.2.1. Steps

Once the deterioration curves for each object are determined, the OIS for the link can be determined. The steps required to do this are given in Table 1. More in-depth explanation of some steps is given in the remainder of Section 2.2. Their exact location is also indicated in Table 1.

2.2.2. Determination of OIS for each object

An IS is used to ensure that an object does not enter CS2 where an adequate level of service would not be provided, may be to execute a preventive intervention at time z and to execute a corrective intervention if the object enters CS2 before time z, i.e. in the period from $(0, z]$. This type of IS is often referred to as age replacement (Gertsbakh, 1997; Gertsbakh, 2000). The explanation of the probability of transitioning from CS1 to CS2 is given in Section 1.

When following this IS, one can envision that impacts are incurred by stakeholders in two ways:

(1) During the execution of interventions ($IC^{u,p}$)[6]: e.g. the owner has to pay for the manual labour required to execute the intervention, and the user has additional travel time due to the required detours.
(2) When the object is in CS2 but the execution of the intervention has not yet begun (SC^{u}): e.g. the owner has to pay for the manual labour required to erect

Table 1. Steps to determine the OIS for the link.

Step	Description	Section
1	Calculate the impacts when an object is in CS2 and no intervention is being executed, SC^u, and the impacts when an intervention is being executed, IC^u.	2.2.2
2	Determine the OIS for each object, e.g. for every t_k^* an intervention should be executed.	2.2.2
3	Determine the time to intervention for each object taking into consideration its actual condition (e.g. optimal intervention time is 10 years, but the object has already been in operation 6 years, then the time to intervention is 4 years).	2.2.2
4	Determine the types of ISs to be investigated.	
5	Determine the values of the reduction factors to be used for each investigated type of IS.	2.2.3
6	Select a type of IS	
7	Determine the time of the next intervention to be executed on the road link, t	
8	Estimate the total impact from time t to the time when the execution of the next intervention.	2.2.4
9	Check $\sum_{k=1}^{K} ic_k^{p,l}(t_k)$, if $\leq B^{p,l}(t_k)$, then all impacts for planned interventions in t_k are within the set limits and therefore all proposed interventions are executed, then move to step 11. Otherwise, move to step 10.	
10	Use priority rule to identify the interventions to be executed. 10.1 Order the objects that are candidates for intervention in decreasing order of contribution to impact incurred due to arriving in CS2 in year t_k. 10.2 Select first object, $k = 1$, $IC^{p,l} = 0$. 10.3 Check $IC_k^{p,l} < B^{p,l}(t_k)$, if yes, object k is selected, go to 10.4. if no, object k is not selected and deferred to next intervention time, go to 10.5. 10.4 Set $IC_k^{p,l} = IC_k^{p,l} + IC^{p,l}$, $B^{p,l}(t_k) = B^{p,l}(t_k) - IC_k^{p,l}$, and $k = k + 1$ and then go to 10.3. 10.5 Set $k = k + 1$ and then go to 10.3.	2.2.5
11	Go to step 7 if the end of the investigated time period has not been reached, otherwise go to step 12.	
12	If all types of ISs have been analysed, select the IS with lowest total impacts as the OIS and go to step 13. If not all types of ISs have been analysed, select another type of IS and go to step 7.	
13	Stop.	

Note: t_k^* is referred to as optimal intervention time for object k, corresponding to z^* in Equation (19). It is also noted that the notation k for object in this section is different from s in Section 2.1, e.g. k can be road section or bridge, while s refers to objects with similar structural characteristics.

signs to reduce the number of lanes in use on a bridge, and the user has additional travel time due to the congestion that this restriction would cause.

This is illustrated in Figure. 2, where τ_A is the time the object enters CS2, τ_B denotes the start of the intervention and τ_C denotes the end of the intervention, where the object is restored to CS1.

IC and SC are given by Equations (11a) and (11b), respectively.

$$IC^{u,p} = \sum_{l=1}^{L} ic_l^{u,p}, \tag{11a}$$

$$SC^u = \sum_{l=1}^{L} sc_l^u, \tag{11b}$$

where $ic_l^{u,p}$ and sc_l^u are the impacts incurred by each stakeholder group $l = (1, \ldots, L)$. The superscripts u and p are referred to as for 'unplanned' and 'planned'. As the same type of intervention is applied in both two cases, it is $ic^{u,p} = ic^u = ic^p$.

Impacts incurred by each stakeholder group l can be estimated by the use of empirical models (Adey, Lethanh, & Lepert, 2012; Kumares & Samuel, 2007). For example, vehicle operation cost during the execution of an intervention can be estimated as a function of daily traffic volume, gasoline unit price, type of vehicle, condition of road, etc. (Kumares & Samuel, 2007). Values of $ic_l^{u,p}$ and sc_l^u can be either positive or negative.

The expected total impact incurred by stakeholders between when an object enters CS2 and when an intervention is started is given by Equation (12). The expected total impact incurred when an object is in CS2

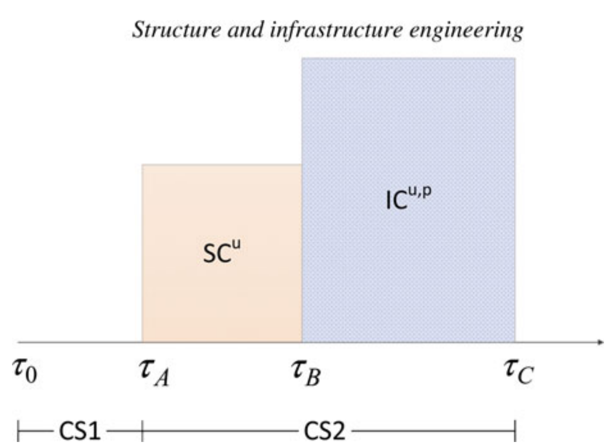

Figure 2. Graphical representation of impact IC and SC.

Table 2. Object information.

Objects	Type	Material	Length (m)	Width (m)	DTV vehicle
1	Road section	Asphalt concrete	290	5.7	600
2	Bridge	Concrete	25	6.5	600
3	Road section	Asphalt concrete	160	5.7	600

and an intervention is being executed is given by Equation (13). The first part of Equation (13) is used to estimate the expected impact due to intervention if the object reaches to CS2. The second part of Equation (13) is used to estimate the expected impact if the object survived until time z and an intervention is executed. Similar formulations can be found in Gertsbakh (1997, 2000) and Lethanh (2009).

$$\mathrm{ESC}(z) = \int_0^z \mathrm{SC}^{\mathrm{u}} \cdot f(t)\exp(-\rho t)\,\mathrm{d}t, \qquad (12)$$

$$\mathrm{EIC}(z) = \int_0^z \mathrm{IC}^{\mathrm{u}} \cdot f(t)\exp(-\rho t)\,dt + \tilde{F}(z)\cdot \mathrm{IC}^{\mathrm{p}}\exp(-\rho z), \qquad (13)$$

where $f(t)$ represents the probability of entering CS2 before a planned intervention is executed and $\tilde{F}(z)$ represents that probability of surviving until the time when a planned intervention is executed. ρ is the discount rate.[7]

In order to determine the OIS, it is necessary to estimate the total impact related to each IS. If an OIS is to be evaluated over a fixed time period, the net present value of total impact $\mathrm{TC}^a(z)$ is given by:

$$\mathrm{TC}^a(z) = \Omega(z) + \sum_{z^\star}\left[\left[\int f(t)\mathrm{TC}^a(z^\star)\,\mathrm{e}^{-\rho t}\,\mathrm{d}t\right]\right] + \mathrm{e}^{-\rho z}\sum_{z^\star}\tilde{F}(z)\,\mathrm{TC}^a(z^\star), \qquad (14)$$

where $\Omega(z) = \mathrm{ESC}(z) + \mathrm{EIC}(z)$ and a is intervention type to be followed. The second and third polynomials of Equation (14) represent the recursive form of Bellman equation in dynamic programming (Bachmann & Konik, 1984; Bellman, 1955; Howard, 1960; Howard, 1971) and represent the expected total impact from the next investigated time interval. As the same type of intervention a will be repeated over and infinite time horizon and expected impact in each interval is considered to be equivalent, it is approximated that $\mathrm{TC}^a(z) = \mathrm{TC}^a(z^\star)$, and therefore, Equation (14) can be expressed as:

$$\mathrm{TC}(0:z) = \int_0^z f(t)\{\mathrm{SC}^{\mathrm{u}} + \mathrm{IC}^{\mathrm{u}} + \mathrm{TC}(0:z)\}\exp(-\rho t)\,\mathrm{d}t + \tilde{F}(z)\{\mathrm{IC}^p + \mathrm{TC}(0:z)\}\exp(-\rho z). \qquad (15)$$

Table 3. Deterioration information.

Objects	Deterioration parameters		CS definition		Other impacts related to entering CS2 (SC^{u}) (mu)
	m	α	CS1	CS2	
1	2.010	0.020	Average roughness of less than 100 mm/m	Average roughness greater or equal to 100 mm/m	20,000
2	2.130	0.003	A wearing index value of less than 0.75	A wearing index value[a] of greater than or equal to 0.75	5000
3	2.020	0.020	Average roughness of less than 100 mm/m	Average roughness greater or equal to 100 mm/m	10,000

[a] Wearing index value represents deterioration of bridge (Brodsky et al., 2006).

Table 4. Intervention information.

Objects	Intervention: Types	Intervention: Effectiveness	Owner impacts related to a planned intervention IC^p (mu)	Owner impacts related to a unplanned intervention IC^u (mu)
1	Resurfacing: resurfacing with 5 cm thickness of asphalt concrete	Restored object to CS1 with 100% probability	72,217	72,217
2	Reconstruction: removal of the concrete cover without exposing the reinforcement and the addition of a new chloride-free concrete cover layer of the same thickness	Restored object to CS1 with 100% probability	10,375	10,375
3	Resurfacing: resurfacing with 5 cm thickness of asphalt concrete	Restored object to CS1 with 100% probability	39,456	39,456

In order to obtain an explicit form of TC, it is necessary to define $\Gamma(z)$ and $\Lambda(z)$ as:

$$\Gamma(z) = \int_0^z f(t)\exp(-\rho t)\,\mathrm{d}t = \int_0^z \alpha m \tau^{m-1}\exp(-\alpha\tau^m - \rho t)\,\mathrm{d}t, \tag{16}$$

$$\Lambda(z) = \tilde{F}(z)\exp(-\rho z) = \exp(-\alpha z^m - \rho z). \tag{17}$$

and substituting these equations into Equation (15) gives:

$$\mathrm{TC}(0:z) = \frac{(\mathrm{SC^u} + \mathrm{IC^u})\Gamma(z) + \mathrm{IC^p}\cdot\Lambda(z)}{1 - \Gamma(z) - \Lambda(z)}. \tag{18}$$

The objective function $\mathrm{TC}(0, z)$ from which the minimum expected total impact of all ISs, and therefore, the OIS is then given by:

$$\mathrm{TC}(0) = \min_z\{\mathrm{TC}(0:z)\}. \tag{19}$$

The OIS, i.e. the optimal interval z^*, is then estimated by taking the first derivative of Equation (18) and by setting it equal to 0, as shown in the following equation:

$$\frac{\mathrm{d}\,\mathrm{TC}(0:z)}{\mathrm{d}z} = \frac{\psi(z)}{\{1 - \gamma(z) - \Lambda(z)\}^2} = 0, \tag{20}$$

where

$$\psi(z) = (\mathrm{SC^u} + \mathrm{IC^u})\Gamma'(z) + \mathrm{IC^p}\cdot\Lambda'(z) + \mathrm{SC^u}\{\Lambda(z)'\cdot\Gamma(z) - \Gamma'(z)\cdot\Lambda(z)\}, \tag{21}$$

$$\Gamma(z)' = \frac{\mathrm{d}\Gamma(z)}{\mathrm{d}z} \quad \text{and} \quad \Lambda(z)' = \frac{\mathrm{d}\Lambda(z)}{\mathrm{d}z}.$$

When $\psi(z) = 0$, the optimal interval z^* has been found. A numerical algorithm to solve Equation (18) is explained in Appendix.

In the determination of the OIS, it is assumed that an intervention is executed when an object is in CS2 and than an intervention restores the object with 100% certainty to CS1.

2.2.3. Determination of the variations in impacts due to the timing of multiple interventions

The determination of the OIS for the road link requires special attention to the variations in impacts due to the timing of multiple interventions, e.g. executing two interventions at the same time is less expensive than executing the same two interventions separately. In this methodology, these impacts are expected when multiple interventions that are simultaneously executed are

Table 5. Investigated IS types.

Strategy type	Objects with interventions executed simultaneously	Objects with interventions executed alone	Reduction factors for objects with simultaneous interventions
1	–	1, 2, 3	
2	1, 2	3	0.84
3	1, 3	2	0.90
4	2, 3	1	0.85
5	1, 2, 3	–	0.90

estimated by multiplying the impacts expected if only one intervention is executed with appropriate reduction factors. For example, if an intervention of type A is expected to cost 10 monetary units (mu) if executed alone, then two interventions of type A would cost 20 mu if executed at entirely different times. If, however, two interventions of type A were to be executed simultaneously and had a reduction factor of 0.9, then they would cost 18 mu (20 × 0.9).

2.2.4. *Determination of intervention candidates*

The time to the next intervention is the minimum of the times to intervention of each object $t_k^*(k \in K)$. The number of objects to be considered in the year of next intervention is

$$(\bar{K} = \arg\min_{k \in K} \{t_k\})$$

.

2.2.5. *Using priority rule*

The determination of the OIS for the road link requires special attention to the constraints on impacts, e.g. limits on the maximum financial resources that can be allocated to all objects or the maximum travel time that can be incurred due to interventions on all objects in a specific time period. These constraints are dealt with in this methodology by implementing a priority rule that selects some interventions to be postponed to future time periods if their simultaneous execution results in the exceedance of an impact constraint. To determine if the value of an impact is exceeded in a time period, the summation over the investigated time period is made of the probabilities of the object being in each CS at each instant of time multiplied with the impact if the object were in that CS over the specific time period. As shown in the following equation for the impacts on the stakeholders:

$$\Delta_k^{\mathrm{u},*}(\bar{t}_k) = \int_t^{t_k - t} \sum_{l=1}^{L} \left(\mathrm{ic}_k^{\mathrm{u},l} + \mathrm{sc}_k^{\mathrm{u},l}\right) f_k(\zeta) \exp(-\rho\zeta)\,\mathrm{d}\zeta, \tag{22}$$

where $f_k(\zeta)$ is the expected failure probability of object k in an elapsed time of $\zeta(\zeta = [t, t_k - t])$. t is the start time of the investigation and t_k is the time of intervention for object k. The terms $\mathrm{ic}_k^{\mathrm{u},l}$ and $\mathrm{sc}_k^{\mathrm{u},l}$ are impacts incurred by stakeholder group l if object k enters CS2.

3. Example

3.1. *Description*

The use of the methodology is demonstrated by determining the OIS for a fictive road link consisted of one bridge and two road sections when there is no budget constraint and when there is a budget constraint of 3500 mu per year. The information related to the object, to deterioration and to the interventions to be executed is given in Tables 2–4, respectively. The investigated ISs and the reduction factors to be used in the estimation of the impacts when multiple interventions are executed simultaneously are given in Table 5. All numbers are fictive but are considered sufficient to demonstrate the methodology. A value of 2% was used as the discount rate, but to investigate the effect of variations in the discount rate the analyses were carried out with discount rates of 6% and 10%. The probabilities of each object staying in CS1 over time (without the execution of an intervention) are shown in Figure 3.

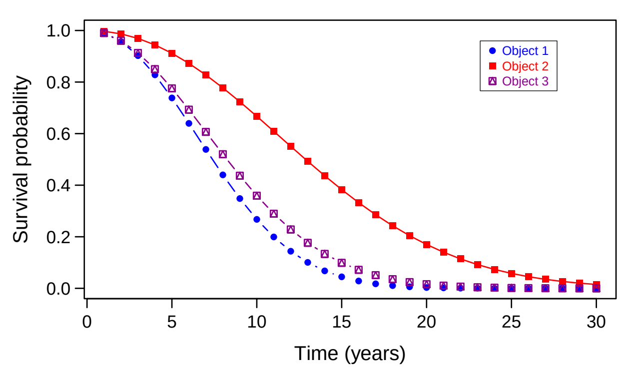

Figure 3. Deterioration.

Table 6. OISs under five strategy types (discount rate 2%).

		No budget constraint				Budget constraint of 3500 mu/year			
Strategy type	Object	Intervention duration (days)	Intervention interval (years)	Average annual impacts (mu)	Reduction in average annual impacts compared to OIS of type 1 (mu)	Intervention duration (days)	Intervention interval (years)	Average annual impacts (mu)	Reduction in average annual impacts compared to OIS of type 1 (mu)
1	1	30	19	3738		30	29	4251	
	2	8	25	205		8	25	205	
	3	15	16	2644		15	16	2644	
	Total			6587	0			7100	0
2	1 + 2	30	23	2916		30	26	3441	
	3	15	16	2644		15	16	2644	
	Total			5560	1027			6085	1015
3	1 + 3	30	17	6140		30	23	6492	
	2	8	25	205		8	26	728	
	Total			6345	241			7220	− 120
4	1	30	19	3739		30	19	3739	
	2 + 3	15	23	1813		15	28	2119	
	Total			5552	1034			5858	1242
5	1 + 2 + 3	30	24	4548		30	25	5152	
	Total			4548	2038			5152	1948

Table 7. OISs under five strategy types (discount rate 6%).

		No budget constraint				Budget constraint of 3500 mu/year			
Strategy type	Object	Intervention duration (days)	Intervention interval (years)	Average annual impacts (mu)	Reduction in average annual impacts compared to OIS of type 1 (mu)	Intervention duration (days)	Intervention interval (years)	Average annual impacts (mu)	Reduction in average annual impacts compared to OIS of type 1 (mu)
1	1	30	21	1033		30	21	1033	
	2	8	29	49		8	29	49	
	3	15	17	763		15	17	763	
	Total			1845	0			1845	0
2	1 + 2	30	26	785		30	26	785	
	3	15	17	763		15	17	763	
	Total			1545	300			1545	300
3	1 + 3	30	19	1681		30	19	1681	
	2	8	29	49		8	29	49	
	Total			1730	115			1730	115
4	1	30	21	1033		30	21	1033	
	2 + 3	15	26	487		15	26	487	
	Total			1520	325			1520	325
5	1 + 2 + 3	30	27	1212		30	27	1212	
	Total			1212	633			1212	633

Table 8. OISs under five strategy types (discount rate 10%).

		No budget constraint				Budget constraint of 3500 mu/year			
Strategy type	Object	Intervention duration (days)	Intervention interval (years)	Average annual impacts (mu)	Reduction in average annual impacts compared to OIS of type 1 (mu)	Intervention duration (days)	Intervention interval (years)	Average annual impacts (mu)	Reduction in average annual impacts compared to OIS of type 1 (mu)
1	1	30	23	520		30	23	520	
	2	8	34	21		8	34	21	
	3	15	18	399		15	18	399	
	Total			940	0			940	0
2	1 + 2	30	30	372		30	30	372	
	3	15	18	399		15	18	399	
	Total			771	169			771	169
3	1 + 3	30	21	839		30	21	839	
	2	8	34	21		8	34	21	
	Total			860	80			860	80
4	1	30	23	520		30	23	520	
	2 + 3	15	30	231		15	30	231	
	Total			751	189			751	189
5	1 + 2 + 3	30	32	575		30	32	575	
	Total			575	365			575	365

3.2. Results

When there is no budget constraint and a discount rate of 2% is used, the OIS executes interventions on all three objects every 24 years for an average impact of 4548 mu/year (Table 2). When there is a budget constraint of 3500 mu/year, the OIS executes interventions on all three objects every 25 years for an average impact of 5152 mu/year. The OISs in both cases are of type 5 and result in savings of 2038 and 1948 mu/year, respectively, when compared to the OISs of type 1. The OISs of each type of ISs under each budget scenario, as well as the intervention duration per object, the intervention interval per object, the average annual impacts and the reduction in average annual impact compared to the OIS of type 1, are given in Table 6.

When there is no budget constraint and discount rates of 6% and 10% are used, the OIS executes interventions on all three objects every 27 years and 32 years, respectively, for an average impact of 1212 and 575 mu/year (Tables 7 and 8). When there is a budget constraint of 3500 mu/year, the OIS in each case is the same. This is because the budget of 3500 mu/year no longer has an effect on the interventions that can be executed in each year. It can be seen by comparing the values given in Tables 6–8 that the use of different discount rates in the range chosen has no effect on the optimality of the ISs but has an effect on the average time between interventions. The use of increasingly higher discount rates increasingly enlarges the average time between interventions.

3.3. Discussion

It can be seen from the example that the proposed Weibull model can be used to determine OISs for road links consisted of multiple objects, allowing for consideration of the past deterioration of each object (deterioration part) and the changes in the impacts that occur due to the timing of multiple interventions (through the reduction factors in the evaluation part).

It can, therefore, be also seen that the investigated methodology can be used to determine OISs on road links that include grouped interventions, and, therefore, interventions on individual objects even if it is not the right time (either earlier or later) according to their ‘own’ OIS. For example, the OIS for the road link (with a 2% discount rate) actually shows that it is better to execute an intervention on object 1 every 24 years even if its ‘own’ OIS indicates that it is best to execute an intervention every 19 years. This means that without consideration of the grouping of interventions, something not done in many existing methodologies, it is in many cases not possible to determine the OIS.

Although the methodology is only demonstrated using a road link consisted of three objects, it can be potentially extended to a road link consisted of many more objects. The main challenges with this are to determine the correct reduction factors and to define the types of ISs that should be investigated in a way that does not result in a combinatorial explosion of the number of OISs (i.e. pro IS type) that need to be determined before the OIS for the road link can be determined.

A potential weakness of the methodology is that it only uses binary states to represent the condition of the objects, making it not possible to distinguish between different types of interventions for each object, something which is done in many existing methodologies, albeit on the single object level. This simplification makes it necessary to make a number of broad approximations which result in a significant loss of information in the determination of optimal strategies, e.g. it is necessary to assume that road users have the same level of service over the entire range of physical condition of a road section between as new and failed. It is believed that models that make it possible to take into consideration more than two CSs may be more suitable for the determination of OISs for road sections affected by gradual deterioration processes. The investigated methodology may be more suitable when attempting to determine OISs for a road link affected by the processes that result in sudden deterioration, such as flooding, where it is more appropriate to view infrastructure objects as fully operational or not fully operational.

4. Conclusions

In this paper, a probabilistic model for the determination of OISs for a road link composed of multiple objects is investigated. In the model, the performance of each object that comprises that link is described by two CSs. The transition time between states is considered as a stochastic variable, which can be represented by a model based on the Weibull hazard function. The parameters of Weibull hazard function are to be estimated using inspection data. The determination of OISs is carried out based on the evaluation of total impacts incurred by all stakeholders. An example was shown to demonstrate the applicability of the methodology.

The main strengths of the methodology were that deterioration history was taken into consideration and that it was possible to consider the execution of interventions simultaneously and therefore associated reductions in impacts that normally occur when interventions are grouped. The main weakness of the methodology was that the condition of objects is represented using only two CSs, i.e. fully operational and not fully operational.

Future work should be focused on the testing of this methodology on real road link with many more objects. This would include the investigation of the sensitivity of the OIS determined using this methodology to changes in the values of the deterioration model parameters and of the impacts associated with interventions.

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Notes

1. Email: adey@ibi.baug.ethz.ch.
2. It is noted at the outset of the paper that the term 'optimal' can be understood as referring to the 'best' IS among possible ones.
3. The function assumed for α can take different forms in regression analysis, e.g. exponential (Lancaster, 1990).
4. Newton method is a method to find the successively better approximations to the roots of a real-valued function.
5. Values of *t*-test should be greater than 1.96 for 95% confidence and the covariance matrix should be non-singular.
6. The superscripts u and p refer to 'unplanned' and 'planned', respectively. As the same type of intervention is selected for both preventive and corrective intervention, it is $IC^{u,p} = IC^{u} = IC^{p}$.
7. The discount rate is a variable used to help balance impacts that occur at different periods of time. The value of the discount rate can greatly affect the present value of a future impact. The proper discount rate should represent the opportunity cost of what else could be accomplished with those same resources (Gruber, 2007). The higher the discount rate, the lower the value of impacts that occur in the future when compared with the value of impacts incurred today. Usually, the higher the discount rate the more likely it is that interventions will be postponed. As small changes in the value of the discount rate can have a large effect on the valuation of ISs, it is normally suggested to include it in appropriate sensitivity analyses before decisions are made.

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Appendix: Solution to Gamma function

Solution to Gamma function in the equation $\Psi(0, z)$

$$\Psi(0:z) = \frac{(\mathrm{SC}+\mathrm{IC})\Gamma(z) + \mathrm{IC}\cdot\Lambda(z)}{1-\Gamma(z)-\Lambda(z)}, \tag{A.1}$$

where $\Gamma(z)$ and $\Lambda(z)$ functions are defined as follows:

$$\begin{aligned}\Gamma(z) &= \int_0^z f(t)\exp(-\rho t)\,\mathrm{d}t \\ &= \int_0^z \alpha m\tau^{m-1}\exp(-\alpha\tau^m - \rho t)\,\mathrm{d}t,\end{aligned} \tag{A.2}$$

$$\Lambda(z) = \tilde{F}(z)\exp(-\rho z) = \exp(-\alpha z^m - \rho z), \tag{A.3}$$

Gamma function in Equation (A.2) can be extended in the following way:

$$\begin{aligned}\Gamma(z) &= \int_0^z (\alpha m\tau^{m-1} + \rho - \rho)\exp(-\alpha\tau^m - \rho t)\,\mathrm{d}t \\ &\Leftrightarrow -\int_0^z \exp(-\alpha t^m - \rho t)\,\mathrm{d}(-\alpha t^m - \rho t) \\ &- \rho\int_0^z \exp(-\alpha t^m - \rho t)\,\mathrm{d}t \\ &= 1 - \Lambda(z) - \rho\int_0^Z \exp(-\alpha t^m - \rho t)\,\mathrm{d}t.\end{aligned} \tag{A.4}$$

The denominator in Equation (A.1) becomes

$$1 - \Lambda(z) - \Gamma(z) = \rho\int_0^z \exp(-\alpha t^m - \rho t)\,\mathrm{d}t. \tag{A.5}$$

Substituting Equations (A.4) and (A.5) into Equation (A.1), the following results are obtained:

$$\begin{aligned}\Psi(0,z) &= \frac{(\mathrm{SC}+\mathrm{IC})[\Gamma(z)+\Lambda(z)-1] + \mathrm{SC} + \mathrm{IC} - \mathrm{SC}\cdot\Lambda(z)}{1-\Lambda(z)-\Gamma(z)} \\ &= \frac{\mathrm{SC}+\mathrm{IC}-\mathrm{SC}\cdot\Lambda(z)}{\rho\int_0^z\Lambda(t)\,\mathrm{d}t} - (\mathrm{SC}+\mathrm{IC}).\end{aligned} \tag{A.6}$$

In order to solve the integration of function $\Lambda(z)$, the general form of expanding the integration into following discrete series is used.

$$I_k = \int_0^{kdt} f(x)\,\mathrm{d}x, \tag{A.7}$$

in which k is the number of iteration and dt is the very small amount of time. For example, value of d can become $d = 0.01$ or 0.001 or even smaller.

$$\begin{aligned}I_{k+1} &= \int_0^{(k+1)dt} f(x)\,dx = I_k + \int_{k.dt}^{(k+1)dt} f(x)\,\mathrm{d}x \\ &= I_k + \frac{[f(kdt) + f\{(k+1)dt\}]\mathrm{d}t}{2}.\end{aligned} \tag{A.8}$$

To this point, the value of integration can be easily estimated by numerical calculation. We substitute Equation (A.6) and use Newton method to estimate the minimum value of $\Psi(0, Z)$ with respect to the increasing number of year Z. Apart from this method, the Simpson rule for solving integration can also be applied. However, a comparison with various small values of d proves that the above method is sufficient enough.