Infrastructure Deterioration Prediction with a Poisson Hidden Markov Model on Time Series Data

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Abstract: The deterioration of a pavement surface can be described in terms of the presence and severity of distinct distresses, like potholes, cracking, and rutting. Each deterioration process is ordinarily described by a set of pavement indicators (e.g., number of potholes, percentage of cracks, international roughness index) that are measured during monitoring and inspection activities. Manifestly, there exist statistical correlations among the deterioration processes. For instance, cracks appearing on a road section may contribute to an increase in pothole occurrence, and vice versa. In order to mathematically formulate the statistical interdependency among deterioration processes, a Poisson hidden Markov model is proposed in this paper. The model describes the complex process of pavement deterioration, which includes the frequent occurrence of local damage (e.g., potholes) as well as the degradation of other pavement indicators (e.g., cracks, roughness). To model the concurrent frequency of local damage, a stochastic Poisson process is used. At the same time, a Markov chain model is used to depict the degradation of other pavement indicators. A numerical estimation approach using Bayesian statistics with a Markov chain Monte Carlo simulation is developed to derive the values of the model's parameters based on historical information. The applicability of the model was demonstrated through an empirical example using data from a Japanese highway road link. **DOI: 10.1061/(ASCE)IS.1943-555X.0000242.** *© 2014 American Society of Civil Engineers.*

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Introduction

In recent years, there has been significant development and application of statistical models in the field of infrastructure asset management. Particularly, in pavement management systems (PMS) and bridge management systems (Madanat and Ibrahim 1995; Anastasopoulos et al. 2011; Kobayashi et al. 2012a), statistical models are preferred due to their capability to incorporate uncertainties inherently embedded in deterioration forecasting and management of civil infrastructure. Among the existing statistical models, Markov models have been widely used. In Markov models, deterioration of civil infrastructure is expressed through transition probabilities between discrete condition states (CS), which are deduced values or composite values of performance indicators (e.g., pavement roughness, cracks on bridge decks) (Thomas and Sobanjo 2013).

Performance indicators provide important information about the physical condition of an infrastructure object. Such information is then used as input for deterioration forecasting models and decision-making models. With current monitoring activities in pavement engineering, performance indicators (pavement distresses) can be measured by high-speed inspection cars equipped with high-definition cameras and sensors. Inspections are scheduled activities that are executed periodically, e.g., once every 2 or 3 years. In connection with inspections, Markov models can be used to predict the deterioration of an infrastructure object and determine optimal intervention strategies for that object and its network (Robelin and Madanat 2007; Kuhn 2012).

Beside the periodic inspections, road administrators are required to perform more frequent patrols. Patrols are needed to promptly detect obstacles appearing on road sections and local damage such as potholes. Potholes can dangerously disturb the riding quality of road users and are therefore either mended on the spot or shortly after being detected. Patrols and inspections are therefore considered as two different monitoring activities, with different objectives. Patrols can also be considered as a type of periodic inspection. However, in this paper, patrols and inspections are treated as two separate activities denoting two fundamentally different kinds of inspection. Patrols are carried out more frequently than inspections. The objective of patrols is not only to detect local damage but also to observe other risk factors, which are not directly associated with the physical condition of roads. Patrols can sometimes be very important and are fundamentally required on a daily or weekly basis. This is particularly true in the case of highway management where vehicles travel at high speed and a high level of safety and riding quality is expected.

From these different monitoring activities, a PMS database includes values of various pavement indicators and local damage for every road section. When inspecting the database, one can see at a glance that deterioration of a road section is not merely a single process but consists of multiple processes. For example, cracks and potholes appearing on a road section can be attributed to two different deterioration processes. Cracks are more manifest and mainly caused by weather, runoff, construction, and maintenance, while the occurrence of potholes could be due to repeated vibration from axle loads and the development of cracks (O'Flaherty 2002). Differences in deterioration processes can be also distinguished by examining the values of pavement indicators and local damage. Cracks are measured as a percentage of cracking area and

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roughness is measured as a continuous value on the international roughness index. The evolution of cracks and roughness over time can be deterministically followed by exponential functions. However, the evolution of potholes (local damage) is quantified in integers; thus, a counting process, such as Poisson distribution, is typically employed.

There are at least two deterioration processes involved in the course of deterioration prediction for a road section: one is the process causing local damage and the other is the process that can be regularly measured through periodic inspections. There is a physical and statistical correlation between the two processes. When cracks appear, the probability of potholes occurring on the same road section might be higher than when the road section has no cracks. This phenomenon has also been mechanistically proved to be true, e.g., a mathematical relationship between cracks and pothole formation was empirically presented in the work of O'Flaherty (2002). The cited author concluded in his work that potholes are not accompanied by distortion of the adjacent surface. They generally result from a cracked bituminous surface, which has allowed moisture to enter and soften the pavement or penetrate horizontally under the bituminous layer.

Decisions regarding pavement repairs (e.g., overlay, crack sealing, patching) are generally made based on important indicators such as the percentage of cracking. Moreover, the cycle of periodic inspection is often 2 to 3 years. Once an inspection has been carried out on a road section, in many cases, the next periodic inspection will not be conducted for a couple of years, mainly due to budget constraints and limitations on resources. On the other hand, potholes generated on road surfaces are detected through daily patrol, and repaired ad hoc by using room-temperature mixtures. However, in road sections where potholes are frequently generated, it is obviously necessary to conduct permanent repair, such as overlaying, rather than ad hoc repair, from the viewpoints of the safety of road users and the cost of road administrators. In practice, when cracking percentage reaches a certain level (e.g., 20% in Japan), overlaying is recommended. However, it is difficult for road administrators to understand the relationship between the frequency of pothole occurrence and the need of overlaying. Therefore, road administrators insist on the necessity to overlay for individual road sections prone to potholes or repeat ad hoc countermeasures until the crack rate reaches 20%, while knowing from their experience that there is a high correlation between potholes and crack rate. If the relation between potholes and crack rate can be evaluated quantitatively with the proposed Poisson hidden Markov model, it will become possible to make decisions about ad hoc or permanent repair, based on the data of pothole generation that is obtained through daily patrols.

To formulate a statistical relation between the two deterioration processes, this study makes the following assumptions:

1. The deterioration process of localized damage such as potholes is modeled with a stochastic Poisson process. This assumption is realistic enough as the Poisson process is appropriately used for modeling count events (count events can also be followed by other stochastic distributions such as negative binomial distribution or zero-inflated distribution). In general, Poisson distribution is easier to work with compared to others, e.g., Poisson distribution has one parameter and negative binomial distribution has two parameters (Kaburagi and Matsumoto 2008; Paroli et al. 2000).
2. The process of surface deterioration is modeled with a Markov model. This assumption is also appropriate as modeling pavement surface with a Markovian approach has been widely used in the field of infrastructure asset management (Madanat and Ibrahim 1995; Robelin and Madanat 2007; Kobayashi et al. 2012a, b; Thomas and Sobanjo 2013).
3. Local damage is monitored in short time intervals (daily, weekly), while pavement indicators such as cracks and roughness are measured periodically, and at much longer time intervals than the monitoring of local damage. Thus, the information on local damage can be used to improve the quality of deterioration prediction with Markov models, and vice versa.

The preceding example of potholes and cracks represents only part of a greater picture that has spurred the formulation of the new mathematical model presented in this paper. At first, the study aimed to develop a hierarchical Markov model that could consider at least two deterioration processes simultaneously. This type of model has been discussed in the field of econometrics, where many authors have tried to estimate the stochastic frontiers of products or cost of business firms based on unobserved heterogeneity factors (Kumbhakar and Lovell 2000). It can also be found in the literature of bioinformatics or pattern and speech recognition in informatics and communication science, where researchers try to use hidden Markov models to reveal the true value of unobserved variables [e.g., computer software detecting words based on vocal voices or speech (Robert et al. 2000; Zucchini and MacDonald 2009)]. Secondly, it is the authors' intention to understand the statistical correlation between the different deterioration processes of civil infrastructure and to make use of available historical data to improve the quality of deterioration prediction. Lastly, the authors are convinced that this new model will motivate research developments where hidden Markov models are used in other transportation sectors (e.g., the prediction of accidents based on a switching regime Markov model; determination of optimal inspection intervals for a road network based on incomplete monitoring data).

In particular, with respect to the first two motivations behind the research work, the new model will be helpful to estimate the overall condition of pavement at any time between infrequent inspections. In this way, infrastructure managers will be able to decide when and where to execute inspections to reveal the true values of pavement indicators. Furthermore, in statistical terms, the overall deterioration curve of a pavement is expected to have higher accuracy in comparison with conventional Markov models using only two sets of data from long-interval inspections, e.g., sometimes the interval between two inspection times is considerably long (4 or 5 years), which impairs the accuracy of the deterioration curve.

In summary, this study proposes a Poisson hidden Markov model to depict the composite deterioration process that is comprised of localized damage and surface deterioration. To model the manifest deterioration of pavement indicators, this study employs the existing Markov model of Kobayashi et al. (2012a). The deterioration process concerning local damage is modeled with a Poisson distribution. In order to obtain the values of the model's parameters, an algorithm was developed based on a Bayesian estimation approach and a Markov chain Monte Carlo (MCMC) simulation. A numerical example was conducted using a data set from Japan to verify the applicability of the model.

Formulation of the Model

Preconditions

Fig. 1 presents a graphical representation of the deterioration processes and observations of local damages and other pavement indicators for a road section. In the figure, polygon boxes and round boxes indicate local damage and other pavement indicators

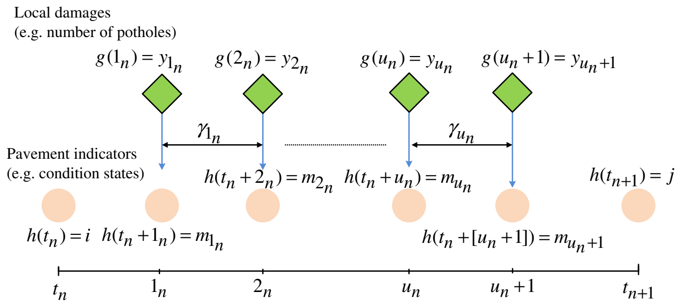

Fig. 1. Scheme of patrols and inspections

(e.g., cracks, roughness), respectively. It is assumed that intervals of inspecting local damage and other pavement indicators are not the same. The local damages are observed more frequently than the other pavement indicators. The values of other pavement indicators, hereafter referred to as *pavement indicators* are periodically measured but with longer interval time between two consecutive inspections. For example, local damages such as potholes are often observed by patrols on a weekly or monthly basis, but inspections to reveal values of cracks and roughness are carried out on a yearly basis.

Along the time axis (Fig. 1), values of pavement indicators are measured periodically at time $t_n (n = 0, 1, \ldots, N)$. For instance, the values at time t_n and t_{n+1} are $h(t_n) = i$ and $h(t_{n+1}) = j$, respectively. It is noted that in this paper, values of pavement indicators $h(t_n) = i$ and $h(t_{n+1}) = j$ are referred to as discrete condition states $[i, j = (1, \ldots, I)$ with I as the absorbing state]. In Markov models, the continuous values of pavement indicators can be transformed to fit a discrete scale. In between an interval τ_n from t_n to t_{n+1} $(\tau_n = t_{n+1} - t_n)$, it is assumed that there is no periodic inspection other than patrols. Patrols are regularly conducted at an interval of $\gamma_{u_n} (u = 0, \ldots, U)$ with the purpose of disclosing any local damage that has occurred on the surface of the road sections. These activities are necessary in many cases in order to reduce exposing risks such as traffic accidents that could suddenly happen due to potholes. Once local damage has been disclosed, it is remedied on the spot or shortly after being detected and the number of local damage cases are registered $[g(u_n) = y_{u_n}]$. The occurrence of local damage is thus a counting process. It is also assumed that at the beginning of an in-service time for a road section, the condition state $h(t_0) = 1$, i.e., the road section is a new one (after construction or interventions). The condition state $h(t_n + u_n) = m_{u_n}$ at any time in between two consecutive periodic inspections (hereafter referred to as *subperiod time* u_n) is not observed by patrols.

If no intervention other than remedies for local damage are executed during the time interval τ_n, the following condition is logical:

$$h(t_n) = \bar{i} \leq \cdots \leq h(t_n + u_n) = m_{u_n} \leq \cdots \leq h(t_{n+1}) = \bar{j} \qquad (1)$$

In Eq. (1), the bar $^-$ over i, j indicates that such discrete values i, j are observable. The use of the bar $^-$ has a similar meaning in subsequent parts of this article. It can be seen both in Eq. (1) and in Fig. (1) that at any time $h(t_{n+u_n})$ within the interval τ_n, the true value of the discrete condition state is not observable. At time t_{n+u_n}, only local damage is disclosed $[g(u_n) = y_{u_n}]$ and registered. To simplify the notation, from this point onward, u and u_n are used interchangeably to denote the subperiod time in (t_n, t_{n+1}).

Markov Transition Probabilities

In a Markov model, an object passes from one CS to another with a certain probability in each unit of time. The Markov transition probability (MTP) are determined using past inspection data, and can be estimated as described in Kobayashi et al. (2012a). An explicit mathematical formula for estimating the MTP p_{ij} can be obtained using data collected at times t_n and t_{n+1}, and by defining the subsequent conditional probability of going from CS i to j, with respect to time interval (t_n, t_{n+1})

$$p_{ij} = \text{Prob}[h(t_{n+1}) = j | h(t_n) = i] = \sum_{l=i}^{j} \prod_{s=i}^{l-1} \frac{\lambda_s}{\lambda_s - \lambda_l} \prod_{s=l}^{j-1} \frac{\lambda_s}{\lambda_{s+1} - \lambda_l} \exp(-\lambda_l \tau_n) \qquad (2)$$

where λ is hazard rate of each condition state; and i, j, s, l are running index of condition state. The hazard rate λ can be described as a function of characteristic variables (e.g., traffic volume, thickness of pavement, bridge slab area, ambient temperature)

$$\lambda_i = \boldsymbol{x\beta}^i \qquad (3)$$

with x and β being vectors of characteristic variables and unknown parameters, respectively.

The matrix of MTP for a period $[t_n, t_n + u]$ can be expressed as $P(u) = \{P\}^u$ and the MTP $p_{ij}(u)$ can be defined as

$$p_{ij}(u) = \sum_{l=i}^{j} \prod_{s=i}^{l-1} \frac{\lambda_s}{\lambda_s - \lambda_l} \prod_{s=l}^{j-1} \frac{\lambda_s}{\lambda_{s+1} - \lambda_l} \exp(-\lambda_l u) \qquad (4)$$

Model

With the earlier assumptions, one can model the occurrence of potholes by a Poisson process, which represents counting events (Madanat and Ibrahim 1995; Paroli et al. 2000). With a Poisson process, the occurrence rate of potholes $\mu(m_u, z_u) > 0$ in the period γ_u is defined as follows:

$$\mu(m_u, \boldsymbol{z}_u) = \boldsymbol{z}_u \alpha^{m_u} \qquad (5)$$

In Eq. (5), $\boldsymbol{z}_u = (z_{1,u}, \ldots, z_{P,u})$ and $\boldsymbol{\alpha}^{m_u} = (\alpha_1^{m_u}, \ldots, \alpha_P^{m_u})'$ are vectors of characteristic variables at subperiod time u within the period τ_n [refer to Fig. (1)] and unknown parameters, respectively. The notation P indicates the numbers of characteristic variables (e.g., traffic volume, pavement thickness).

Statistically, the occurrence rate $\mu(m_u, \boldsymbol{z}_u)$ can be expressed as a conditional probability defined in following equation:

$$\pi(y_u | m, z_u) = \text{Prob}[g(u) = y_u | h(t_n + u) = m, z_u] = \exp[-\mu(m, \boldsymbol{z}_u)] \frac{[\mu(m, \boldsymbol{z}_u)]^{y_u}}{y_u!} \qquad (6)$$

In Eq. (6), the notation m is used instead of m_u for better visualization of the mathematical form. Without loss of generality, in subsequent parts of the paper, m_u and m will be used interchangeably in some mathematical polynomials for ease of reading. This equation infers that the probability of observing y_u potholes at subperiod time u is dependent on the probability that the discrete condition state $h(t_{n+u}) = m$. In other words, there exists statistical correlation between the numbers of potholes that might occur and the underlying true condition state of pavement indicators. Also, in Eq. (6), the sum of probabilities of all events must equal 1, i.e., $\sum_{y_u=0}^{\infty} \pi(y_u | m) = 1$. In addition, the probability $\rho_u(m|i)$ of observing $h(t_{n+u}) = m$ at time t_{n+u} is also a conditional probability of the event that its value is i at time $h(t_n)$

$$\rho_u(m|i) = \text{Prob}[h(t_n + u) = m | h(t_n) = i] = p_{i,m}(u) \qquad (7)$$

Based on Eqs. (6) and (7), one can define the probability $\tilde{\pi}_u(y_u, z_u)$ that guarantees the event y_u numbers of potholes have been occurred within a subperiod time interval γ_u

$$\tilde{\pi}_u(y_u, z_u) = \sum_{m=i}^{j} \pi(y_u|m_u, z_u)\rho_u(m_u|i) \tag{8}$$

Here, we can estimate the likelihood $\mathcal{L}(\bar{\xi}_n, \boldsymbol{\theta})$ of having all observed information $\bar{\xi}_n = \{\boldsymbol{y}_u, z_n, \bar{i}, \bar{j}\}$ in a time period $\tau_n = [t_n, t_{n+1}]$. This information vector includes a vector representing the numbers of potholes ($\bar{\boldsymbol{y}}_u$) occurring by subperiod time u and a vector concerning the true value of condition states ($\bar{i}$, $\bar{j}$) observed at time t_n and t_{n+1}. In recursive form, the likelihood function $\mathcal{L}(\bar{\xi}_n, \boldsymbol{\theta})$ is expressed as follows:

$$\mathcal{L}(\bar{\xi}_n, \boldsymbol{\theta}) = \pi(\bar{y}_0|\bar{i}, z_0) \sum_{m_1=i}^{j} p_{\bar{i},m_1} \ell_1(m_1) \tag{9a}$$

$$\ell_u(m_u) = \pi(\bar{y}_u|m_u, z_u) \sum_{m_{u+1}=m_u}^{j} p_{m_u,m_{u+1}} \ell_{u+1}(m_{u+1}) \tag{9b}$$

$$(1 \le u_w \le T_n - 1)\ell_{U-1}(m_{U-1}) = \pi(\bar{y}_{U-1}|m_{U-1}, z_{U-1}) p_{m_{U-1},\bar{j}} \tag{9c}$$

The vector of unknown parameter $\boldsymbol{\theta}$ is comprised of α and $\boldsymbol{\beta}$, which are defined in Eqs. (3) and (5), respectively.

Estimation Method

Data Set

In the management of a highway road network, data are collected for individual road sections. It is assumed that there are K road sections. The index of each road section is $k = (1, \ldots, K)$. Thus, the observed data set of a road section k can be generalized as a vector $\bar{\xi}_n^k = [\boldsymbol{y}_u^k, z_n^k, \bar{h}^k(t_n), \bar{h}^k(t_{n+1})]$. Eventually, a total set $\bar{\Xi} = \{\bar{\xi}_n^k: n = 1, \ldots, N, k = 1, \ldots, K\}$, which includes the panel data of all road sections in different time intervals, is used to construct a full likelihood functional form $\mathcal{L}(\bar{\Xi}, \boldsymbol{\theta})$

$$\mathcal{L}(\bar{\Xi}, \boldsymbol{\theta}) = \prod_{k=1}^{K} \prod_{n=1}^{N} \mathcal{L}(\bar{\xi}_n^k, \boldsymbol{\theta}) \tag{10}$$

Up until this point, it can be concluded that the objective of our estimation is to estimate the value of parameter vector $\hat{\boldsymbol{\theta}}$ that maximizes the likelihood function in Eq. (10).

The likelihood function of the model involves high-order nonlinear polynomials, with a large number of solutions for the first-order optimality conditions (Robert et al. 2000). With such a likelihood function it is not feasible to use the conventional maximum likelihood estimation (MLE) approach. In order to obtain the parameter values of the model, it is suggested in Bayesian statistics that a complete likelihood function must be derived using posterior distributions of models parameters (Geman and Geman 1984; Jeff 2006).

Complete Likelihood Function

For a road section k, its condition states observed at $h(t_n)$ and $h(t_{n+1})$ are $\bar{i}_n^k$, $\bar{j}_{n+1}^k$. During the period $\tau_n = (t_n, t_{n+1})$, the representation of the condition state at the respective subperiod time u is a vector $\boldsymbol{m}_n^k = (m_1^k, \ldots, m_{U-1}^k)$. Eq. (1) thus can be rewritten as

$$\bar{i}_n^k \le m_1^k \le \cdots \ m_u^k \ \cdots \le m_{U-1}^k \le \bar{j}_{n+1}^k \tag{11}$$

This vector is assumed to represent the true condition state of the road section at subperiod time u. However, it is unmeasurable and thus remains hidden. As we aim to uncover the hidden condition state, the word hidden condition states is therefore excluded. Instead, the term *potential variable* $\boldsymbol{m}_n^k = (\tilde{m}_1^k, \ldots \tilde{m}_u^k \ldots, \tilde{m}_{U-1}^k)$ is used, with $\sim$ representing the possibility that it must be predicted. In addition, the dummy variable $\delta_{s_u}^k$ is introduced to specify the condition that the potential variable must be satisfied

$$\delta_{s_u}^k = \begin{cases} 1 & \tilde{m}_u^k = s_u^k \\ 0 & \tilde{m}_u^k \ne s_u^k \end{cases}, \quad (s_u^k = \bar{i}_n^k, \ldots, \bar{j}_n^k; u = 1, \ldots, U-1)$$

With the introduction of the potential variable $\boldsymbol{m}_n^k$ and the dummy variable $\delta_{s_u}^k$, the likelihood function defined in Eqs. (9a)–(9c) can be rewritten as follows:

$$\begin{aligned} \bar{\mathcal{L}}(\boldsymbol{m}^k, \bar{\xi}_n^k, \boldsymbol{\theta}) &= \prod_{u=0}^{U-1} \prod_{s_{u+1}^k=\bar{i}^k}^{\bar{j}^k} \pi^k(\bar{y}_u|s_u^k, \bar{z}_u^k)^{\delta_{s_u}^k} \{p_{s_u^k, s_{u+1}^k}\}^{\delta_{s_u}^k} \\ &= \prod_{u=0}^{U-1} \pi^k(\bar{y}_u|\tilde{m}_u^k, \bar{z}_u^k) p_{\tilde{m}_u^k, \tilde{m}_{u+1}^k} \end{aligned} \tag{12}$$

In Bayesian statistics, the likelihood function defined in Eq. (12) is often referred to as the *complete likelihood function* (Jeff 2006). Using this function, it is possible that a full conditional posterior distribution of the potential variable $m_u^k = m[m \in (\tilde{m}_{u-1}^k, \ldots, \tilde{m}_{u+1}^k)]$ can be estimated

$$\begin{aligned} \text{Prob}(m_u^k = m|\boldsymbol{m}_{-u}^k) &= \frac{\bar{\mathcal{L}}(\boldsymbol{m}_{-u}^k, \bar{\xi}_n^k, \boldsymbol{\theta})}{\sum_{m=\tilde{m}_{u-1}^k}^{\tilde{m}_{u+1}^k} \bar{\mathcal{L}}(\boldsymbol{m}_{-u}^k, \bar{\xi}_n^k, \boldsymbol{\theta})} \\ &= \frac{\pi^k(\bar{y}_u^k|m, \boldsymbol{z}_u^k)\omega_{m,t}^k(\tilde{m}_{u-1}^k, \tilde{m}_{u+1}^k)}{\sum_{m=\tilde{m}_{u-1}^k}^{\tilde{m}_{u+1}^k} \pi^k(\bar{y}_u^k|m, \boldsymbol{z}_u^k)\omega_{m,t}^k(\tilde{m}_{u-1}^k, \tilde{m}_{u+1}^k)} \end{aligned} \tag{13}$$

where

$$\boldsymbol{m}_{-u}^k = (\tilde{m}_1^k, \ldots, \tilde{m}_{u-1}^k, \tilde{m}_{u+1}^k, \ldots, \tilde{m}_U^k),$$
$$\boldsymbol{m}_{-u}^{m,k} = (\tilde{m}_1^k, \ldots, \tilde{m}_{u-1}^k, m, \tilde{m}_{u+1}^k, \ldots, \tilde{m}_U^k), \quad \text{and}$$

$$\omega_{m,t}^k(\tilde{m}_{u-1}^k, \tilde{m}_{u+1}^k) = \begin{cases} p_{\bar{j}_n,m}^k p_{m,\tilde{m}_2}^k & u = 1 \\ p_{\tilde{m}_{u-1},m}^k p_{m\tilde{m}_{u+1}}^k & 2 \le u \le U-2 \\ p_{\tilde{m}_{U-2},m}^k p_{m,\bar{j}_n}^k & u = U-1 \end{cases} \tag{14}$$

Apparently, if one can obtain the occurrence probability of potholes $\pi^k(\bar{y}_u^k|m, \bar{z}_u^k)$ and the MTP $p_{m,\tilde{m}_{u+1}^k}^k$ $(u = 0, \ldots, U-1;$ $n = 0, \ldots, N, k = 1, \ldots, K)$, one can derive the full conditional posterior distribution of condition state $m_u^k \in \{\tilde{m}_{u-1}^k, \ldots, \tilde{m}_{u+1}^k\}$ at the subperiod time u given the condition state $\boldsymbol{m}_{-u}^k$.

Using the MCMC method, which takes advantage of the full conditional posterior probability [Eq. (27)], the potential variable $\boldsymbol{m}$ is generated randomly, and the parameters α, $\boldsymbol{\beta}$ are estimated. The use of the MCMC method is further described in subsequent sections.

Estimation Algorithm

MCMC Method

The numerical estimation of parameter values of mixture distribution models in general, and of the proposed model in particular, is not feasible with conventional MLE methods since the mathematical setup of the models results in a high-order nonlinear likelihood function (Robert et al. 2000; Kobayashi et al. 2012a). To overcome the limitation of the MLE approach, Bayesian statisticians recommend the use of the MCMC method. This section describes a MCMC algorithm developed to obtain the parameter values of the proposed model.

First, it is necessary to define conjugate prior distributions for unknown parameters α^i and $\boldsymbol{\beta}^i$. This paper hypothesizes that the prior distributions for the two parameters follow a multivariate normal distribution, i.e., $\alpha^i \sim \mathcal{N}_P(\boldsymbol{\zeta}^{i,\alpha}, \boldsymbol{\Sigma}^{i,\alpha})$ and $\boldsymbol{\beta}^i \sim \mathcal{N}_Q(\boldsymbol{\zeta}^{i,\beta}, \boldsymbol{\Sigma}^{i,\beta})$. The probability density functions (PDF) of $\boldsymbol{\alpha}^i$ and $\boldsymbol{\beta}^i$ are respectively shown in Eqs. (15) and (16):

$$\phi(\boldsymbol{\alpha}^i|\boldsymbol{\zeta}^{i,\alpha}, \boldsymbol{\Sigma}^{i,\alpha}) = \frac{1}{(2\pi)^{P/2}\sqrt{|\boldsymbol{\Sigma}^{i,\alpha}|}} \times \exp\left[-\frac{1}{2}(\boldsymbol{\alpha}^i - \boldsymbol{\zeta}^{i,\alpha})(\boldsymbol{\Sigma}^{i,\alpha})^{-1}(\boldsymbol{\alpha}^i - \boldsymbol{\zeta}^{i,\alpha})'\right] \tag{15}$$

$$\psi(\boldsymbol{\beta}^i|\boldsymbol{\zeta}^{i,\beta}, \boldsymbol{\Sigma}^{i,\beta}) = \frac{1}{(2\pi)^{Q/2}\sqrt{|\boldsymbol{\Sigma}^{i,\beta}|}} \times \exp\left[-\frac{1}{2}(\boldsymbol{\beta}^i - \boldsymbol{\zeta}^{i,\beta})(\boldsymbol{\Sigma}^{i,\beta})^{-1}(\boldsymbol{\beta}^i - \boldsymbol{\zeta}^{i,\beta})'\right] \tag{16}$$

In Eqs. (15) and (16), the signs $\boldsymbol{\zeta}$ and $\boldsymbol{\Sigma}$ represent the prior expected value of the parameters and prior variance covariance matrix.

A complete posterior PDF is then defined as

$$\rho(\boldsymbol{\alpha}, \boldsymbol{\beta}|\boldsymbol{m}, \boldsymbol{\xi}) \propto \mathcal{L}(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{m}, \boldsymbol{\xi}) \prod_{i=1}^{I-1}\{\phi(\boldsymbol{\alpha}^i|\boldsymbol{\zeta}^{i,\alpha}, \boldsymbol{\Sigma}^{i,\alpha})\psi(\boldsymbol{\beta}^i|\boldsymbol{\zeta}^{i,\beta}, \boldsymbol{\Sigma}^{i,\beta})$$
$$\propto \prod_{k=1}^{K}\prod_{n=1}^{N}\prod_{u_n=0}^{T_n-1}\left[\exp(-z_{u_n}^k \alpha^{\tilde{m}_{u_n}^k})(z_{u_n}^k \alpha^{\tilde{m}_{u_n}^k})^{\bar{y}_{u_n}^k} \cdot \Gamma\right] \cdot \Lambda \tag{17}$$

where

$$\Gamma = \sum_{l=\tilde{m}_u^k}^{\tilde{m}_{u+1}^k}\left\{\prod_{h=\tilde{m}_u^k}^{l-1}\frac{\lambda_h^k}{\lambda_h^k - \lambda_l^k}\prod_{h=l}^{\tilde{m}_{u+1}^k}\frac{\lambda_h^k}{\lambda_{h+1}^k - \lambda_l^k}\exp(-\lambda_l^k)\right\} \tag{18}$$

$$\Lambda = \prod_{i=1}^{I-1}\exp\left[-\frac{1}{2}(\boldsymbol{\alpha}^i - \boldsymbol{\zeta}^{i,\alpha})(\boldsymbol{\Sigma}^{i,\alpha})^{-1}(\boldsymbol{\alpha}^i - \boldsymbol{\zeta}^{i,\alpha})' - \frac{1}{2}(\boldsymbol{\beta}^i - \boldsymbol{\zeta}^{i,\beta})(\boldsymbol{\Sigma}^{i,\beta})^{-1}(\boldsymbol{\beta}^i - \boldsymbol{\zeta}^{i,\beta})'\right] \tag{19}$$

Gibbs Sampling

Gibbs sampling (Geman and Geman 1984) is a powerful MCMC algorithm that has been widely used in Bayesian statistics. This paper applies the Gibbs sampling algorithm to obtain the value of the posterior PDF $\rho(\boldsymbol{\alpha}, \boldsymbol{\beta}|\boldsymbol{m}, \boldsymbol{\xi})$ and then extracts samples of parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

The following steps briefly describe the Gibbs sampling algorithm:

Step 1: Select Initial Values

1. Randomly select the value of prior distribution [Eqs. (15) and (16)] parameter vector (matrix) $\boldsymbol{\zeta}^{i,r}, \boldsymbol{\Sigma}^{i,r} (i = 1, \ldots, I-1; r = \alpha, \beta)$;
2. Select initial values of potential variable $\boldsymbol{m}^{(0)} = [\boldsymbol{m}_n^{k(0)}: k = 1, \ldots, K; n = 1, \ldots, N]$, $\boldsymbol{m}_n^{k(0)} = [\tilde{m}_1^{k(0)}, \ldots, \tilde{m}_{U_n-1}^{k(0)}]$ under the condition defined in Eq. (11);
3. Randomly select initial values of parameters $\boldsymbol{\alpha}^{(0)}$ and $\boldsymbol{\beta}^{(0)}$; and
4. The influence of initial values grows weaker in proportion to the increasing numbers of the MCMC simulations. The number of MCMC samples v is $v = 1$.

Step 2: Extract Samples of $\boldsymbol{\alpha}^{(v)}$

Based on the value of potential variable $\tilde{\boldsymbol{m}}^{(v-1)}$ defined in the previous step, in this step the parameter values associated with the occurrence rate of potholes $\boldsymbol{\alpha}^{(v)} = [\boldsymbol{\alpha}^{1(v)}, \ldots, \boldsymbol{\alpha}^{I-1(v)}]$, $\boldsymbol{\alpha}^{i(v)} = [\alpha_z^{i(v)}: z = 1, \ldots, P]$ can be derived. The Gibbs sampler used in this step is then defined in conjunction with the complete conditional posterior density function $\rho[\boldsymbol{\alpha}^{(v)}|\boldsymbol{m}^{(v-1)}, \boldsymbol{\xi}]$

$$\hat{\rho}[\boldsymbol{\alpha}^{i(v)}|\boldsymbol{m}^{(v-1)}, \boldsymbol{\xi}] \propto \prod_{k=1}^{K}\prod_{n=1}^{N}\prod_{u_n=0}^{U_n} \times \left\{\exp\left[-z_{u_n}^k \alpha^{\tilde{m}_{u_n}^{k(v-1)}(v)}\right]\left[z_{u_n}^k \alpha^{\tilde{m}_{u_n}^{k(v-1)}(v)}\right]^{\bar{y}_{u_n}^k}\right\}^{\delta_i^{u_n k}} \times \exp\left\{-\frac{1}{2}[\boldsymbol{\alpha}^{i(v)} - \boldsymbol{\zeta}^{i,\alpha}](\boldsymbol{\Sigma}^{i,\alpha})^{-1}[\boldsymbol{\alpha}^{i(v)} - \boldsymbol{\zeta}^{i,\alpha}]'\right\} \tag{20}$$

where $\delta_i^{u_n k}$ is a dummy variable governed by the following conditions:

$$\delta_i^{u_n k} = \begin{cases} 1 & \text{when } \tilde{m}_{u_n}^k = i \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

Furthermore, the unknown parameter α_p^i can be defined conditionally on α_{-p}^i (p is an index number in a set of P components).

$$\hat{\rho}[\alpha_p^i|\boldsymbol{\alpha}_{-p}^i, \boldsymbol{m}^{(v-1)}, \boldsymbol{\xi}] \propto \prod_{k=1}^{K}\prod_{n=1}^{N}\prod_{u_n=0}^{U_n} \times \left\{\exp\left[-z_{p,u_n}^k \alpha_p^{\tilde{m}_{u_n}^{k(v-1)}(v)}\right]\left[z_{u_n}^k \alpha^{\tilde{m}_{u_n}^{k(v-1)}(v)}\right]^{\bar{y}_{u_n}^k}\right\}^{\delta_i^{u_n k}} \times \exp\left[-\frac{\sigma_{pp}^i}{2}(\alpha_p^i - \hat{\zeta}_p^i)^2\right] \tag{22}$$

$$\hat{\zeta}_p^i = \zeta_p^i + \sum_{h=1,\neq p}^{P}(\alpha_h^h - \zeta_h^i)\sigma_{hp}^i \tag{23}$$

In Eq. (23), values of α_{-p}^i and $\tilde{\boldsymbol{m}}^{(v-1)}$ are known from the previous step. ζ_p^i is the number pth component of the prior expectation vector $\boldsymbol{\zeta}^i$, and σ_{hp}^i is the number (h, p) component of prior distribution variance-covariance matrix $\boldsymbol{\Sigma}^{i-1}$. Moreover, $\sum_{h=1,\neq p}^{P}$ is the sum total of components from 1 to P, excluding p. Here, $\alpha^{(v)} = [\alpha_1^{1(v)}, \ldots, \alpha_Q^{I-1(v)}]$ is randomly sampled with the following procedures:

1. Step 2-1 Randomly generate $\hat{\rho}[\alpha_1^{1(v)}|\boldsymbol{\alpha}_{-1}^{1(v-1)}, \boldsymbol{m}^{(v-1)}, \boldsymbol{\xi}]$ from $\alpha_1^{1(v)}$.
2. Step 2-2 Randomly generate $\hat{\rho}[\alpha_2^{1(v)}|\boldsymbol{\alpha}_{-2}^{1(v-1)}, \boldsymbol{m}^{(v-1)}, \boldsymbol{\xi}]$ from $\alpha_2^{1(v)}$.
3. Step 2-3 Repeat these steps.

4. Step 2-4 Randomly generate $\hat{\rho}[\alpha_P^{I-1(v)}|\boldsymbol{\alpha}_{-P}^{I-1(v-1)}, \boldsymbol{m}^{(v-1)}, \boldsymbol{\xi}]$ from $\alpha_P^{I-1(v)}$.

It is noted that in the algorithm an adaptive rejection sampling method (Gilks and Wild 1992) is used to sample parameter $\boldsymbol{\alpha}$ from Eq. (23).

Step 3: Extract Samples of $\boldsymbol{\beta}^{(v)}$

In this step, samples are extracted of parameter $\boldsymbol{\beta}^{(v)}$. The Gibbs sampler $\hat{\rho}[\beta_q^e|\boldsymbol{\beta}_{-q}^e, \boldsymbol{m}^{(v-1)}, \boldsymbol{\xi}]$ of β_q^e, when $\boldsymbol{\beta}_{-q}^e$ is already known, is defined by

$$\hat{\rho}[\beta_q^e|\boldsymbol{\beta}_{-q}^e, \boldsymbol{m}^{(v-1)}, \boldsymbol{\xi}] \propto \prod_{i=1}^{i}\prod_{j=i}^{I}\prod_{k=1}^{K}\prod_{u_n=1}^{U_n-1} \times \left[\prod_{l=i}^{j-1}(\beta_q^i x_q^k)^{\delta_{ij}^{u_n^k}-\delta_{ie}^{u_n^k}} \sum_{h=i}^{j}\prod_{l=i}^{h-1}\frac{\lambda_l^k}{\lambda_l^k-\lambda_h^k}\prod_{l=h}^{j-1}\frac{\lambda_l^k}{\lambda_{l+1}^k-\lambda_h^k}\exp(-\lambda_h^k)\right]^{\delta_{ij}^{tk}} \times \exp\left[-\frac{\sigma_{qq}^e}{2}(\beta_q^e-\hat{\zeta}_q^e)^2\right] \tag{24}$$

where

$$\hat{\zeta}_q^e = \zeta_q^e + \sum_{h=1,\neq q}^{Q}(\beta_h^e-\zeta_h^e)\sigma_{hq}^e \tag{25}$$

and $\delta_{ie}^{u_n k}$ and $\delta_{ij}^{u_n k}$ are dummy variables satisfying the following conditions:

$$\delta_{ie}^{u_n k} = \begin{cases} 1 & \text{when } \tilde{m}_{u_n}^k = i = e \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

$$\delta_{ij}^{u_n k} = \begin{cases} 1 & \text{when } \tilde{m}_{u_n}^k = i, \tilde{m}_{u_n+1}^k = j \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

and ζ_q^e is the q^{th} component of the prior expectation vector $\boldsymbol{\zeta}^e$, and σ_{hq}^e is the number (h, q) component of the prior distribution variance-covariance matrix $\boldsymbol{\Sigma}^{e-1}$.

Step 4: Update Potential Variable

Now random samples can be generated of the new potential variable $\boldsymbol{m}^{(v)}$ based on full conditional posterior probability [Eq. (13)]. Here, the potential variable vector is defined as $\boldsymbol{m}_{-u_n}^{k(v)} = [\tilde{m}_1^{k(v)}, \ldots, \tilde{m}_{u_n-1}^{k(v)}, \tilde{m}_{u_n+1}^{k(v)}, \ldots, \tilde{m}_{T_n-1}^{k(v)}]$. At this time, the full conditional posterior probability of $m_{u_n}^{k(v)}\{m_{u_n}^{k(v)} \in [\tilde{m}_{u_n-1}^{k(v)}, \ldots, \tilde{m}_{u_n+1}^{k(v)}]\}$ is

$$\text{Prob}[m_{u_n}^{k(v)} = m|\boldsymbol{\alpha}^{(v)}, \boldsymbol{m}_{-u_n}^{k(v-1)}, \boldsymbol{\xi}] = \frac{\pi^k\left[\bar{y}_{u_n}^{k(v)}\middle|m, z_{u_n}^k\right]\omega_{m,u_n}^k\left[\tilde{m}_{u_n-1}^{k(v)}, \tilde{m}_{u_n+1}^{k(v-1)}\right]}{\sum_{m=\tilde{m}_{u_n-1}^{k(v)}}^{\tilde{m}_{u_n+1}^{k(v-1)}}\pi^k\left[\bar{y}_{u_n}^{k(v)}\middle|m, z_{u_n}^k\right]\omega_{m,u_n}^k\left[\tilde{m}_{u_n-1}^{k(v)}, \tilde{m}_{u_n+1}^{k(v-1)}\right]} \tag{27}$$

For every $k(k = 1, \ldots, K); n = 1, \ldots, N$, the potential variable $\tilde{m}_{u_n}^{k(v)}(u_n = 1, \ldots, U_n)$ is obtained, from $u_n = 1$.

Step 5: Decide to End Algorithm

In this step, values of parameters $\boldsymbol{\alpha}^{(v)}, \boldsymbol{\beta}^{(v)}$ are recorded, as well as the value of the potential variable $\tilde{\boldsymbol{m}}^{(v)}$. If $v < \bar{v}$, then $v = v + 1$, and the program will return to Step 2. If not, the algorithm ends.

Moreover, in order to eliminate the bias that could potentially be embedded in the obtained values of parameters and potential variables, all sampling values generated in a certain initial number of iterations will be dropped. This practice is recommended in Bayesian statistics (Geweke 1996; Jeff 2006). In Bayesian statistics, samples are generated in hundreds or thousands of iterations until their values converge to stable states (Geweke 1996). Values of samples in initial iterations are removed, e.g., if there are 10,000 iterations, values of the first 3,000 iterations are removed.

Finally, the remaining generated values of the model's parameters and potential variable are checked with the Geweke (1996) method, which is a standard test extensively used in Bayesian statistics to verify the significance of estimation results.

Example

Overview

The model was tested with a numerical example using a data set of a two-lane highway link in Japan. The data set was collected over a period from August 2007 to September 2011. The link is located in an urban area and it has a heavy daily traffic volume (DTV), i.e., approximately from 17,000 to 35,000 vehicles per day. Records in the data set were stored in 1,671 sections. Each road section belongs to one of three road types (earthwork-embankment, earthwork-cut, and bridge). The majority of the sections are of the earthwork types (more than 90%). Data on pavement indicators such as cracks, roughness, and unevenness were measured on a yearly basis. Data on potholes were recorded every time a patrol was executed. On average, the duration between pothole occurrences is about 450 days. There were 366 total potholes that were distributed on 143 road sections. A summary of the data is shown in Table 1.

Within the investigated period, there were in total 366 potholes. This number includes potholes that occurred at the same location at different patrol times. At a glance, it was realized from the data set that the number of potholes on sections with heavier DTV were significantly higher than on sections with less DTV. In addition, the number of potholes were also different with respect to road types. As can be seen from Table 1, more potholes were observed in the cruising lane than in the passing lane.

To apply the model on the data set, the study transformed the continuous value of cracks into five discrete condition states (CSs) (Table 2), with CS 1 representing no cracks and CS 5 including the cases when the percentages of cracks on a section was greater than or equal to 10%. It is important to note that the CS can be a composite index that includes several indicators (Talvitie 1999). This example selected only the crack as an indicator to demonstrate the applicability of the model. In addition, the choice of using only five condition states, with their thresholds, was rationally based on the fact that in Japan, if percentages of cracks on the surface of a highway section get to be more than 10%, the road section

Table 1. Overview of Data

Road structure	Bridges	Earthwork (embankment)	Earthwork (cut)	Total
Lane (cruising)	117	162	20	299
Lane (passing)	20	23	24	67
Total	137	185	44	366

Table 2. Definition of Condition States

Conditionstates	Crack percentage (Cr%)
$i = 1$	Cr = 0
$i = 2$	0 < Cr < 2.5
$i = 3$	2.5 ≤ Cr < 5
$i = 4$	5 ≤ Cr < 10
$i = 5$	10 ≤ Cr

is considered to be in bad state. Thus, a scale of 5 is good enough to represent how a road deteriorates with respect to cracks (Kaito et al. 2007).

Estimation Results

Following the steps described in the section "Estimation Algorithm," a numerical calculation was performed with 15,000 iterations of the Gibbs sampler and the MCMC simulation. Convergence of parameter values was reached after the first 5,000 iterations, indicating the model's robustness. Estimation results were obtained along with their statistical tests for parameters β_i [Eq. (3)] and α_i [Eq. (5)], hazard rate $E[\lambda_i]$ of each CS i (crack), expected duration $E[RMD]$, and occurrence rate of potholes $E[\mu_i]$.

Expected Values of β_i

In Table 3, values of parameters β_i are shown along with their statistical test values (confidence intervals and Z-score values of the Geweke test). $\beta_{i,1}$ and $\beta_{i,2}$ represent a constant term and traffic volume, respectively. In addition, the expected hazard rate $E[\lambda_i]$ and the duration of staying in each CS i are also given. The test was performed for samples of the last 10,000 iterations (the first 5,000 samples were dropped to eliminate bias). The obtained Z-score values for all parameters are lower than 1.96, which compares well with the estimation results. If the value of Z-score is less than 1.96, it means the convergent hypothesis cannot be dismissed at a significant level of 5% (Geweke 1996).

Using Eq. (3), expected values of hazard rates $E[\lambda_i]$ were calculated. The obtained values of hazard rates were then used in Eq. (2) to derive the values of the MTP (Table 4). The MTP values in Table 4 were estimated for 1 month. It can be seen that values of the MTP in the diagonal cells of the MTP matrix are very high (more than 0.98) compared to that of the other cells. There are cases in which values of the MTP between two CSs were close to 0, inferring an extremely small probability of transition. Because of these reasons, this study used only three digits for the values of the MTP in Table 4, and that explains why there are some cells with 0 values.

After values of the hazard rates λ_i and the m.t.p π_{ij} are estimated, deterioration curves (Fig. 2) and the distribution of CSs (Fig. 3) over time can be drawn. It is noted that in order to draw the deterioration curves in Fig. 2, the authors separately performed the calculation of the model on three different levels of heavy DTV. The average DTV was 24,184 vehicles/day, while the maximum and minimum value of heavy DTV were set to be 35,153 and 17,049 vehicles/day, respectively. The purpose of having three different DTV scenarios was to understand the influence of the DTV on the deterioration (hazard rate) of the pavement sections.

It can be concluded from Fig. 2 that under average DTV it takes about 37 years for a road section to reach CS 5 from CS 1. The deterioration speed of cracking is fastest from CS 1 to CS 2 (about 7 years), then it becomes slower from CS 2 to CS 3 (more than 10 years). From CS 3 to CS 4 and from CS 4 to CS 5, the deterioration speed becomes a bit faster than that of CS 2 to CS 3. In the case of having the maximum DTV, the deterioration speed tends to be two times faster than in the case of having average DTV (it takes almost 17 years to reach CS 5 from CS 1). In a similar pattern, if the DTV is reduced to a minimum value, the deterioration speed of a road section becomes about twice as slow as the average one (it takes about 73 years to reach CS 5 from CS 1). The deterioration pattern of a road section can be further seen in the distribution of CSs (Fig. 3). After about 3.5 years in service, there were about

Table 3. Parameter Values Associated with Markov Properties

Condition states	Constant terms (β_{i1})	Traffic volume (β_{i2})	Hazard rate [$E(\lambda_i)$]	Duration [E(RMD)]
$i = 1$	−4.801	1.066	—	—
	(−5.064, −4.612)	(0.769, 1.467)	0.017	4.869
	−0.020	0.014	—	—
	−6.408	1.821	—	—
$i = 2$	(−6.698, −6.045)	(1.274, 2.239)	0.006	14.441
	−0.107	0.112	—	—
	−5.785	2.175	—	—
$i = 3$	(−6.193, −5.346)	(1.511, 2.766)	0.014	6.074
	0.139	−0.138	—	—
	−8.944	5.895	—	—
$i = 4$	(−9.640, −8.245)	(5.007, 6.744)	0.008	11.067
	−0.024	0.024	—	—

Note: In each cell, the first value is the expected value of the parameter, the second values in parentheses are the confidence interval, and the last value is the Z-score value of the Geweke test.

Table 4. Markov Transition Probabilities

Prior CSs	Posterior condition states $i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
$i = 1$	0.984	0.016	0.000	0.000	0.000
$i = 2$	0.0	0.993	0.007	0.000	0.000
$i = 3$	0.0	0.0	0.986	0.014	0.000
$i = 4$	0.0	0.0	0.0	0.994	0.006
$i = 5$	0.0	0.0	0.0	0.000	1.000

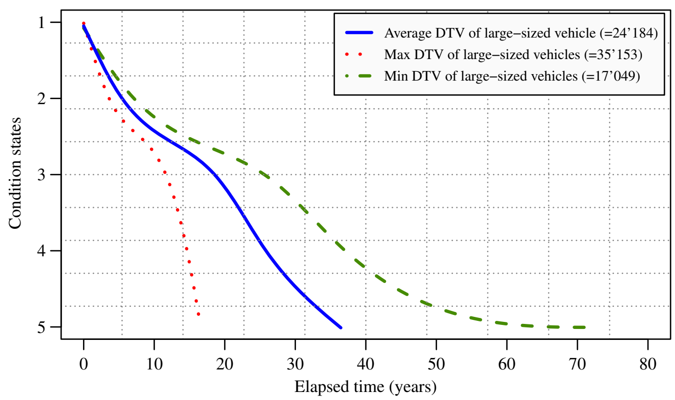

Fig. 2. Deterioration curves (cracking)

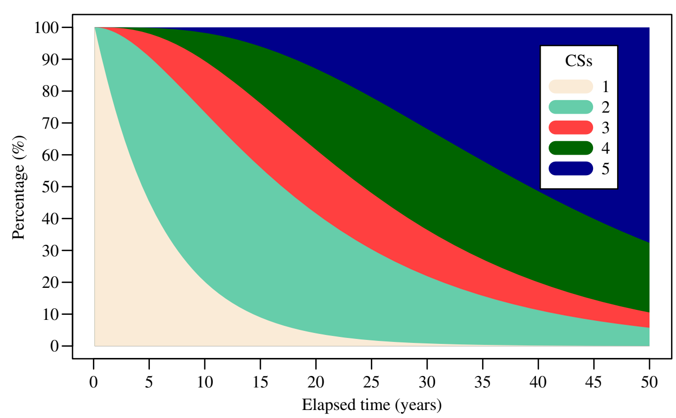

Fig. 3. Condition states distribution

50% of road sections (or fraction of a road section) entering CS 2 from CS 1, and after about 33 years in service, 50% of road sections reach CS 5.

Expected Values of α_i

The expected occurrence rate of potholes on each condition state was also estimated along with the expected value of β_i. The results are summarized in Tables 5–7 and in the graphs in Figs. 4–6.

Table 5 shows the values of parameter α_i, which is the occurrence rate of potholes within each CS i. In this example, there are five CSs; thus, the occurrence rates of Poisson distribution function were defined for each CS i (α_i). Table 5 also gives the values of confidence intervals and Z-score values of the Geweke test. It can be concluded from the parameter values α_i that the higher the CSs, the greater the values of occurrence rates. In other words, the more frequently that cracks occur, the more frequently do potholes occur. This conclusion can be further confirmed by interpreting the curves in Fig. 4. The figure shows the cumulative occurrence probability for one pothole being observed on two different road sections having a bridge structure (cruising lane and passing lane) of 100 m in 2-year periods. After 2 years, the probability of pothole occurrence on the cruising lane tends to be three times higher than on the passing lane.

The relation between the number of potholes and its occurrence probability in each CS within 1 year after the immediate repairs was also investigated and the results are illustrated in Fig. 5 along with the distributions of the probability densities for each number of

Table 5. Parameter Estimation Results

Condition states	Constant terms (α_{i1})	Cruising = 1, passing = 0 (α_{i2})	Bridge = 1, earthwork = 0 (α_{i3})	Occurrence rate $[E(\mu_i)]$
	−9.763	4.190	0	—
$i = 1$	(−10.629, −8.986)	(3.427, 5.089)	—	0.0038
	−0.016	0.004	—	—
	−7.054	0.357	1.927	—
$i = 2$	(−7.290, −6.825)	(0.052, 0.659)	(1.638, 2.211)	0.0085
	−0.014	0.105	−0.036	—
	−8.342	2.180	2.284	—
$i = 3$	(−9.109, −7.668)	(1.490, 2.959)	(1.793, 2.757)	0.0207
	0.136	−0.131	0.032	—
	−10.569	5.615	2.576	—
$i = 4$	(−11.474, −9.477)	(4.539, 6.527)	(2.215, 2.919)	0.0927
	−0.015	0.013	0.008	—
	−8.017	2.107	3.739	—
$i = 5$	(−9.190, −6.863)	(0.965, 3.300)	(2.988, 4.453)	0.1140
	0.142	−0.149	−0.051	—

Note: For each CS, the first row is the expected value of the parameter, the second row the lower and the upper bounds of the 90% confidence interval of the estimated parameter, and third row the Geweke test statistics.

Table 6. Probability Density of Pothole Occurrence after 1 Year (Bridge)

	Numbers of potholes			
Lane	0	1	2	3
Cruising	0.255	0.358	0.248	0.139
Passing	0.846	0.141	0.012	0.001

Table 7. Expected Numbers of Potholes (1-km Section, Cruising Lane, Bridge)

Time (months)	Condition states				
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
3	0	1	2	11	14
	(0.456)	(1.018)	(2.482)	(11.129)	(13.677)
6	0	1	2	8	10
	(0.342)	(0.767)	(1.862)	(8.347)	(10.258)
9	0	1	1	6	7
	(0.228)	(0.509)	(1.241)	(5.565)	(6.839)
12	0	0	1	3	3
	(0.114)	(0.255)	(0.621)	(2.782)	(3.419)

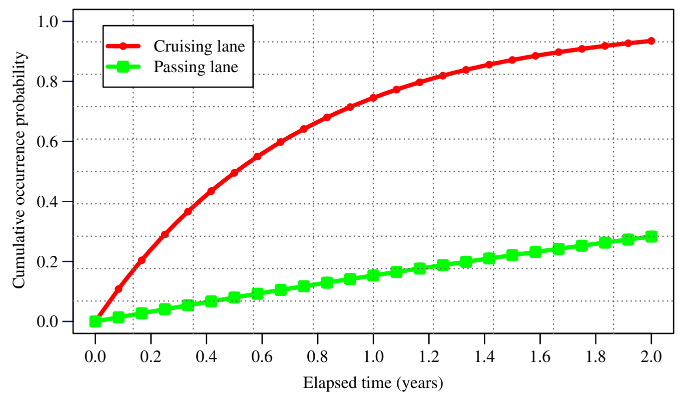

Fig. 4. Cumulative occurrence probability (by lane, bridge)

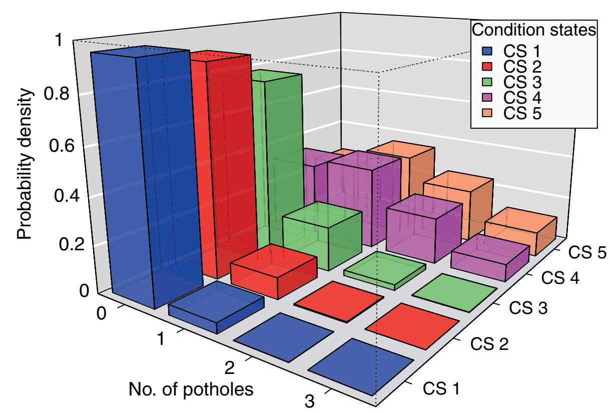

Fig. 5. Poisson distribution by condition states (after 1 year, cruising lane, bridge)

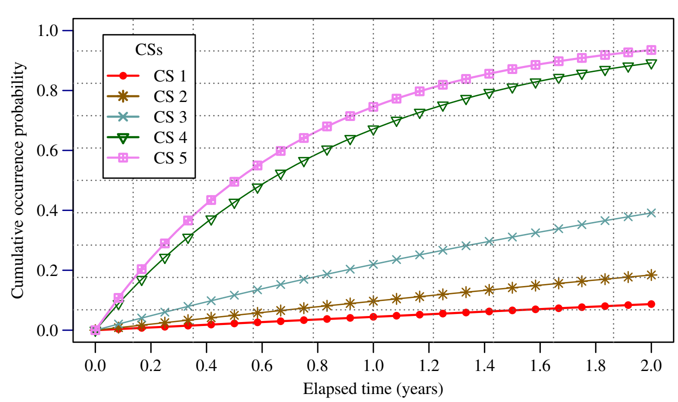

Fig. 6. Cumulative occurrence probability for each CS (cruising lane, bridge)

potholes (0, 1, 2, and 3 potholes). The distributions of probability densities of the occurrence rates of potholes at each CS in the figure were calculated for the cruising lane of the road sections having a bridge structure. If the road section is at CS 1, the probability of having no pothole within the first year is about 0.96 and the probability of having one, two, and three potholes are significantly smaller (about 4.3×10^{-3}, 9.0×10^{-4}, and 1.5×10^{-5}, respectively). On the other hand, if the percentage of cracks increases (the index of CS increases), the probability of one or more potholes occurring is also increased.

Table 6 further explores the relation between the probability of potholes occurring in different lanes when they are at CS 5. A significant difference can be seen in the probability of potholes occuring between the cruising lane and passing lane. As a conclusion, potholes tend to occur more in the cruising lane than in the passing lane.

Similar to Fig. 4, Fig. 6 shows the curves corresponding to the cumulative occurrence probability of having one pothole in the cruising lane of a road section having a bridge structure within a period of 2 years for each different CS. Evidently, the longer the time the road section is in service, the higher the probability of potholes occurring, and the worse the CS is, the higher the probability of potholes occurring becomes. For instance, at CS 5, the probability reaches 0.5 after only 213 days in service. From this result, it cannot be concluded appropriately that the durability of immediate repairs last approximately 213 days when the percentage of cracks on the road section is more than 10%. The cumulative occurrence probability of potholes of 0.5 partially implies that in about half of the numbers of the road sections, potholes had occurred more than once. The value of the cumulative occurrence probability could become an important indicator for management. For example, at the value of 0.1 of the cumulative occurrence probability, the expected number of days to have one pothole when the road section at is CS 5, CS 4, CS 3, CS 2, and CS 1 is 30, 61, 183, 395, and 850 days, respectively.

Table 7 shows the expected number of potholes within 1 year after immediate repairs on a road section of 1 km length. The numbers are given as integers after rounding off from their estimated values shown in the brackets. Taking CS 1 as an example, it can be observed that after 6 months, the expected value of the number of potholes is close to 0, but the value keeps increasing in higher CSs, i.e., at CS 3 and CS 4, the values are 1.241 and 5.565, respectively. In other words, after 1 year, if one pothole was observed on the 1-km section of road, the CS of the surface condition is likely to be $i = 3$; however, if more than five potholes are observed, the CS of the entire section tends to progress to $i = 4$. In such a situation, the entire road section could be considered in a heavily damaged state; consequently, intervention should be recommended rather than performing routine immediate repairs for potholes only.

Conclusions

This paper has presented a Poisson hidden Markov model that can be used to predict deterioration processes on pavement structures. The model enables us to predict simultaneously two deterioration processes of road sections: one process is the occurrence of local damage (e.g., potholes) and the other is the manifest deterioration of pavement indicators that could be represented by discrete condition states. The two processes are statistically interdependent and they can be mathematically formulated.

The model was empirically tested with an example using a data set of a highway link in Japan. It was found in the estimation results that as the road surface condition deteriorates, the probability of having potholes occurring on the surface of highway sections tends to increase. For instance, values of occurrence rate of potholes if the section is in CS 5 are about 2.5 times larger than for sections classefied as CS 1.

The authors aim to introduce a new class of Markov models to the field of infrastructure asset management. Besides the benefit that the quality of deterioration prediction becomes more accurate, the authors also see a great potential for employing different classes of hidden Markov models to solve other problems in the transportation engineering sector, such as the estimation of accident risk associated with pavement condition or the determination of the optimal inspection interval.

It is expected that future research extensions will include the following topics: (1) testing the applicability and the usefulness of the model on a wide range of data, especially data in different regions or nations; (2) a comparative study using the proposed model and other state-of-the-art models in deterioration prediction to further illustrate the robustness and applicability of each model; (3) the use of other stochastic distributions such as the negative binomial distribution or the zero-inflated distribution instead of the Poisson distribution to fit with different behaviors in the data (e.g., data with overdispersion between its mean and variance); and (4) the consideration of unobserved heterogeneity embedded in deterioration processes [e.g., using mixture hazard models for addressing unobserved heterogeneity factors (Lancaster 1990)].

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