Journal of Civil Engineering and Management

ISSN: 1392-3730 (Print) 1822-3605 (Online) Journal homepage: http://www.tandfonline.com/loi/tcem20

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To cite this article: Nam Lethanh & Bryan T. Adey (2016) A real option approach to determine optimal intervention windows for multi-national rail corridors, Journal of Civil Engineering and Management, 22:1, 38-46, DOI: 10.3846/13923730.2014.994030

To link to this article: http://dx.doi.org/10.3846/13923730.2014.994030

Published online: 18 Dec 2015.

JOURNAL OF CIVIL ENGINEERING AND MANAGEMENT
ISSN 1392-3730/eISSN 1822-3605
2016 Volume 22(1): 38–46
doi:10.3846/13923730.2014.994030

A REAL OPTION APPROACH TO DETERMINE OPTIMAL INTERVENTION WINDOWS FOR MULTI-NATIONAL RAIL CORRIDORS

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Received 18 Oct 2013; accepted 15 Sep 2014

Abstract. In this paper, a real option approach to determine the optimal time to execute interventions on rail infrastructure, when it is not known for certain which intervention is to be executed, is presented (i.e. the optimal intervention window). Such an approach is useful in the management of rail infrastructure that belongs to a multi-national rail corridor where multiple railway organizations, ideally, should coordinate their maintenance interventions, years in advance, to minimize service disruptions. The approach is based on an adaptation of the Black and Scholes differential equation model used to value European call options in financial engineering. It is demonstrated by determining the optimal intervention window for infrastructure in a fictive rail corridor.

Keywords: maintenance, rail infrastructure, real options.

Introduction

In order to ensure that rail infrastructure provides an adequate level of service it is necessary that rail managers execute interventions to counter the deterioration caused by use and by slow environmental deterioration process, such as the corrosion of the reinforcement in concrete bridges. Since the execution of interventions normally results in the disruption of service it is beneficial to try to group interventions in a way to minimize this disruption. When multiple rail managers are involved, as is the case for multi-national rail corridors (e.g. Prorail in the Netherlands, DB in Germany, SBB and BLS in Switzerland and RFI in Italy for the Rotterdam-Genoa freight corridor) it is therefore useful for all involved organizations to agree on time to execute interventions, even if it is not known which type of intervention, if any will be executed at that time. This would most likely improve the coordination of interventions on multiple parts of the corridor, which would result in both lower intervention costs and lower service disruptions (Higgins *et al.* 1999; Grimes, Barkan 2006).

Given the inherent uncertainty associated with the conditions of rail infrastructure (e.g. tracks, bridges), an intervention window must be fixed in the future without knowing exactly the types of interventions to be executed on the infrastructure components. This decision will only be made as the time of intervention draws close and more condition information is known. Previous work has indicated that this is possible using a real options approach, such as proposed by Zhao and Tseng (2003) and Santa-Cruz and Heredia-Zavoni (2011) in other civil engineering applications.

Previous research on the use of real option approaches has been focused mainly on the construction of new infrastructure. Some examples include: the construction of a new airport (Smit 2003), of a high-speed passenger train system (Pimentel *et al.* 2012), of an electricity distribution network (Agusdinata 2005), and of a new addition to an existing highway network (Zhao *et al.* 2004) or in the development construction projects in general (Ford *et al.* 2002). There has been little research on the use of real option approaches for the maintenance of existing infrastructure and none was found for the maintenance of multi-national rail corridors. For maintenance of existing infrastructure (Santa-Cruz, Heredia-Zavoni 2011) provided an example of how a real options approach could be used for the maintenance of offshore structures. Although no published research was found on the use of real options approaches in the maintenance of rail infrastructure, there has been on planning optimal maintenance interventions for rail infrastructure (Zoeteman 2001; Patra 2009; Pimentel *et al.* 2012).

The real options approach presented in this paper is based on an adaptation of the Black and Scholes differential equation model used to value European call options in financial engineering (Black, Scholes 1973). It is demonstrated by determining the optimal intervention window for infrastructure in a fictive rail corridor.

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Although certainly not without administrative hurdles, the fixing of optimal intervention windows (OIW) for freight corridors, or parts of freight corridors, would move countries in a direction that would reduce intervention costs and service disruptions increasing the competitiveness of rail in Europe. This is in line with Directive 2012/34/EU (2012) which aims to reduce problems associated with co-ordination and establish a single European railway area. Substantial coordination has already occurred with respect to scheduling trains on and operating European rail corridors (EUROCOM 2008; OECD 2005; Ghijsen *et al.* 2007; DG-MOVE 2011).

The remainder of this paper is consists of five additional sections. Section 1 contains a literature review, in which the limitations of the state-of-the-art for maintenance of rail infrastructure are discussed. Section 2 contains the formulation of the Black and Scholes (1973) real option model used. Section 3 and 4 contain an example and a sensitivity analysis on the results to illustrate how the methodology can be used and how sensitive it is to variations in the value of numerous key parameters, respectively. The last section contains the conclusions and recommendations.

1. Background

Due to the development of computerized decision support systems (Zoeteman 2001; Crozet 2004; Caetano, Teixeira 2013), there has been an increasing focus in research on:

- The development of probabilistic models to improve the prediction of future infrastructure condition, e.g. the prediction of the deterioration of rails (Zhang *et al.* 2013), switches (Kaewunruen, Remennikov 2008), ballasts and sleepers (Zhao *et al.* 2006), and track geometry (Guler *et al.* 2011).
- The determination of the reliability, availability, maintainability, and safety of rail infrastructure objects (Lyngby *et al.* 2008; Rhayma *et al.* 2013; Macchi *et al.* 2012), as well as for networks (Caetano, Teixeira 2013; Macchi *et al.* 2012; Podofillini *et al.* 2006).
- Determine optimal time to intervene on infrastructure. Examples of the later includes the determination of optimal intervention strategies for track maintenance on a railway network (Zhang *et al.* 2013; Zoeteman 2001), for ballast tamping and renewal (Zhao *et al.* 2006).

Although all of this work, if implemented, will help rail managers decide when to intervene on rail infrastructure, it can only indirectly be used to determine the optimal time to intervene if it is not known what intervention is to be executed; something of utmost importance when determining OIWs for multi-national rail corridors, to minimise costs and service disruptions.

By using a real option approach it is possible to determine the OIW. Real option approaches were initially developed in the field of financial engineering, but have been increasingly used in the evaluation of engineering problems (Santa-Cruz, Heredia-Zavoni 2011; Smit 2003; Chiara *et al.* 2007). It is often said that real option approaches are often used to value the flexibility of systems to adapt to uncertain changes of demand. In our case the system being investigated includes the infrastructure and the rail managers, whereas the flexibility of the rail managers is being incorporated directly into the problem. The flexibility is considered to have a value, which is similar to the value of an option in the field of finance. This value varies as a function of the volatility of the values of uncertain parameters, e.g. the condition of the tracks and the price of fuel, and the discount factor, that affect which decision will be made, e.g. the type of intervention to be executed.

2. Real option model

The model used in the real option approach, herein referred to as the real option model, is developed to determine an OIW during a finite time period, T, for an existing railway link. The OIW is considered to occur at time z. At time z the rail manager will decide which type of intervention will be executed. The types of intervention include the “do nothing” intervention. The rail manager is interested in determining the intervention window that will maximize total expected net benefits, i.e. the optimal z on interval $[0, T]$. Net benefits are the difference between the money obtained from operating the rail link minus the costs due to routine maintenance, operation, and more substantial interventions, i.e. the ones executed at z.

The net benefit varies over time due to fluctuations in all of these. It may decrease due to deterioration of the infrastructure which may lead to increases in the number of service disruptions, e.g. due to switch failures, which need to be repaired immediately and may result in financial penalties. It may increase due to increases in the amount of goods to be shipped across the rail link.

Some of the lowest net benefits per time unit (e.g. per month or per year) will be incurred in the time unit of execution of an intervention at z, whereas some of the highest net benefits per time unit will be incurred in the time immediately following the execution of an intervention, e.g. it is expensive to replace existing tracks with new ones, but the number of smaller realignments in the subsequent time units will be drastically lower than before.

In the real option model used here, is similar to the so called “European call option” in financial engineering, where at a predetermined time z, the holder of an option is allowed to make a decision on whether the option will be exercised, but not the obligation to do so (Zhao *et al.* 2004; Hull 2011).

Following table lists notations used in the mathematical formulation of the RO model (see Table 1).

Table 1. Notations

Symbol	Meaning
$B(t)$	Revenue – operating costs in time t
C	Cost to execute an intervention of type d
$N_i(t)$	Quantity of transported goods i to time t
$O(t)$	Operating cost to time t
$R(t)$	Reliability of the rail link, which is calculated through the reliability of the objects in the link.
$S(t)$	Annual profit to time t
W_j	Cost of type j when an adequate level of service is not provided
T	Investigated time period
d	Intervention type
d^*	Reference intervention type (e.g. do-nothing)
$h_i(t)$	Price of transported good i
z	Time to execute an intervention $z \in [0, T]$
t	Running index of time $t \in [0, T]$
ρ	Discount factor (e.g. interest rate)
$\rho_{i,h}$	Standard deviation of the change in the value of $h_i(t)$
μ_i	Drift parameter
$\omega_{i,t}$	Parameter used to model uncertainty of $h_i(t)$ using Wiener process with zero mean and standard deviation of $\sigma_{i,h}\sqrt{dt}$.

In our case, the objective function is to maximize net benefit:

$$S(0:T) = \int_0^z S(0,t)e^{-\rho t}dt + e^{-\rho \cdot z}\int_z^T S^{d^*}(t,T)e^{-\rho t}dt + \text{Max}\left\{e^{-\rho z}\int_z^T\left[S^d(t,T) - S^{d^*}(t,T)\right]e^{-\rho t}dt, 0\right\}. \quad (1)$$

In Eqn (1), the first term represents net benefit from time 0 until the end of the time in which an intervention is executed, the second term represents the net benefit from the end of the time in which an intervention is executed until the end of the investigated time period if the "do nothing" intervention is executed, and the third term represents the difference in net benefits from the end of the time in which an intervention is executed until the end of the investigated time period.

The superscript d denotes intervention type ($d \in D$), where do nothing is also seen as an intervention type and D is a set of intervention types[1].

It is implied in the third term of Eqn (1) that if the execution of intervention d is more beneficial than the execution of intervention d^*, i.e. that its execution will result in higher net benefits, that intervention d will be executed. Otherwise, intervention d^* will be executed.

The net benefit $S^d(t)$ is given by:

$$S^d(t) = B^d(t) - C^d(t); \quad (2)$$

If $t = z$, $C^d(t) = C^d(z)$, otherwise, it equals to 0.

The total net benefit when a rail manager has the possibility at z to decide whether or not to execute intervention d and the decision strategy in which there is the possibility at z to decide whether or not to execute intervention d^*, (denoted as Δ) at time 0 (analogous to the payoff in European call option) is given by:

$$\Delta\left[z, S(z,T)\right] = e^{-\rho(T-z)}\mathbb{E}g\left(Z_T^{z,S^d(t)}\right), \quad (3)$$

where: $\mathbb{E}g\left(Z_T^{z,S^d(t)}\right)$ is the expected value of:

$$g(x) = Max\left[S^d(z,T) - S^{d^*}(z,T), 0\right], \quad (4)$$

where the function $g(x)$ represents an abstract representation of the third polynomial in Eqn (1).

The solution for Eqn (3) has been extensively described in numerous references on applying Black and Scholes formulation (Hull 2011; Iacus 2011) and a summary of it is given in the Appendix of the paper. The total net benefit can be determined using the following equations:

$$\Delta = S(z,T)\Phi(d_1) - e^{-\rho(T-z)}S^{d^*}(z,T)\Phi(d_2), \quad (5)$$

with:

$$d_1 = d_2 + \sigma\sqrt{T-z}; \quad (6)$$

$$d_2 = \frac{\ln\left(\dfrac{S^d(z,T)}{S^{d^*}(z,T)}\right) + \left(\rho - \dfrac{1}{2}\sigma^2\right)(T-z)}{\sigma\sqrt{T-z}}, \quad (7)$$

where: $\Phi(x)$ is the cumulative distribution function for normal standard distribution:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{\frac{-s^2}{2}} ds. \quad (8)$$

S is calculated in each time unit as:

$$S(t) = B(t) - C(t),$$

where:

$$B(t) = \sum_{i=1}^{I}\left\{R(t)\left[N_i(t)\cdot h_i(t) - O_i(t)\right] - (1-R(t))\cdot W_i\right\} \quad (9)$$

and $C(t)$ takes the value of 0 if no intervention is executed. This is analogy to Eqn (2) but use for any time units with and without intervention as an explicit form. Explanations of each of the variables are given in Table 1.

Here, without loss of generality, the superscript and subscript are ignored.

The values of each of these variables can be modeled as uncertain. In many cases, they can be modeled as a geometric Brownian motion (Hull 2011). For example, if the revenue per amount of transported goods is uncertain:

$$dh_i(t) = \mu_{i,h}\cdot h_i(t)\cdot dt + \sigma_{i,h}\cdot h_i(t)\cdot d\omega_{i,t}. \quad (10)$$

[1] The intervention considered here is the intervention on the link and includes the interventions to be executed all objects of the link.

The expected value of uncertain variables given the value at time t, shown here using $h(t)$, is given by:

$$E[h_i(t+u)] = h_i(t) \cdot e^{\mu_i \cdot u}, \tag{11}$$

where u is the length of time between t to $t + u$.

It is assumed that the value of T is chosen so that the salvage value of the objects or the condition of the objects at T can be neglected.

In order to find the optimal z (as a variable of Eqn (1)), i.e. the OIW, it is required to solve the set of Eqns (3) to (8). These equations, which involve the integral of an embedded stochastic process, can be solved using the analytical and numerical approach suggested by Black and Scholes (1973) and Itō (1951). Although these equations can be solved to determine the OIW directly, they can also be used to run simulations to illustrate how the "value" of the intervention window changes with its timing. This intervention window value corresponds with the "option value" used in financial engineering. The entire process of calculation can be simplified in following flowchart (Fig. 1).

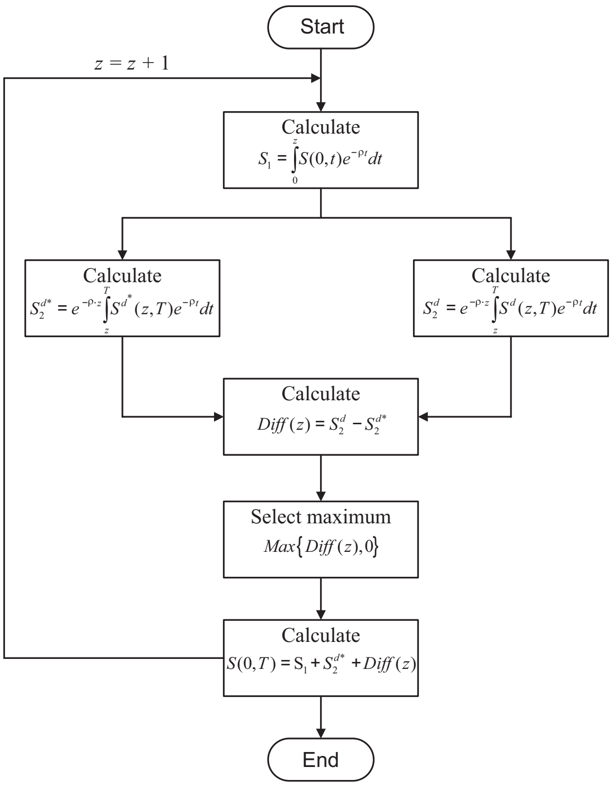

Fig. 1. Flowchart of the RO model

In the flowchart, equations shown in the boxes correspond to the explicit mathematical form shown in Eqn (1). $Diff(z)$ is used to represent the Max{} function of the third polynomial. The intervention window with the highest value is the OIW.

3. Example

3.1. Description

The real option approach is demonstrated for a possible future situation where the rail managers of a trans-European rail corridor are asked to decide on a period of time where they will reduce the traffic on the corridor and during this time execute sufficient preventive interventions so that they guarantee service until the end of the investigated time period. This situation is one that is likely with the increased integration required by the European Union (Directive 2012/34/EU 2012; EUROCOM 2008; OECD 2005). The example is fictive but realistic. It is a high level example to ensure comprehension without being lost in technical details. The intervention window is to be fixed immediately following an agreement to fix an intervention window.

It is assumed that the rail link can be represented as consisting of 10 sections[2], which could be further divided. It is assumed that they reliability of each section can be modeled using the Weibull function with the values of the scale and shape parameters shown in Table 2.

Table 2. Values of reliability parameters

Section	Before intervention		After intervention	
	Scale	Shape	Scale	Shape
	λ	*k*	λ	*k*
1	128	3.264	122	2.652
2	125	3.200	120	2.600
3	115	2.944	110	2.392
4	131	3.360	126	2.730
5	129	3.296	124	2.678
6	124	3.168	119	2.574
7	138	3.520	132	2.860
8	119	3.040	114	2.470
9	121	3.104	116	2.522
10	133	3.424	128	2.782

The reliability of the link, in each time unit (*tu*), is then given by:

$$R(t) = \prod_{m=1}^{10} r_m(t) = \prod_{m=1}^{10} \exp\left(-\frac{t}{\lambda_m}\right)^{k_m}. \tag{12}$$

In Eqn (12), m represents index of each rail section in a link. The parameters λ and k are scale parameter and shape parameter of the Weibull function used to model the change in reliability over time. Values of these parameters can be estimated using historical data (Macchi *et al.* 2012).

The interventions to be executed in the intervention window are substantial maintenance interventions, e.g. replacing the tracks. Routine maintenance, e.g. track

[2] A rail section includes rail track components.

alignment, is done on a regular basis and being considered as a part contributing to the operating cost. It is assumed that the amount of routine maintenance required increases as a function of time since the start of the investigated period and the last intervention window. These increasing costs are expressed as through decreasing reliability of the link. For example, the longer the time since the start of the investigated time period the higher the probability that the tracks will need a realignment that is considered to be routine maintenance. Each realignment costs the same amount of money. In other words, the rail link has an increasing failure rate. Failure here of course means that an adequate level of service is not provided and not necessarily something catastrophic.

Once the intervention window arrives the rail manager has to decide which intervention to execute. In this example it is assumed that the two possible intervention types are:

– A renewal intervention, where the link is restored in a way that its reliability, immediately following the intervention is 100% and starts to decrease again in following time units, and
– Do nothing.

This decision is then made dependent on the value of the uncertain variables when the intervention window arrives. In this case, when the intervention window is in *tu* 46 the amount that can be charged to transport goods must be above 21 *mus*.

It is assumed that the interventions to be executed will take no more than one unit of time. The task of the rail manager is to determine the optimal intervention window within an 80 *tus* time period. It is assumed that the reliability of the rail link at the end of the 80th *tus* is not important and therefore no salvage value is required in the analysis.

The values of all variables used in the model are given in Table 3.

Table 3. Model's input parameters

Symbol	Value	Unit
C_d	4×10^6	*mus*
$N(t=0)$	3×10^5	*units*
$O_d\,(t)$	5×10^5	*mus*
T	80	*tus*
W	5×10^5	*mus*
d	Renewal	*NA*
$d\,*$	Do-nothing	*NA*
$h(t=0)$	15	*mus*
ρ	2	%
σ	0.2	*NA*
μ	0.0003	*NA*

Note: *mus* stands for monetary units; tus stands for time units; subscript i and j shown in Table 1 are omitted for general representation (e.g. one type of good N and one type of cost W incurred).

It is important to note that there is no attempt in this paper to suggest the values of discount rate, drift parameters, the standard deviations in the Wiener process, or the values of reliability of a rail link to trigger intervention. The values selected are only to chosen to allow an illustrative example to be conducted.

3.2. Results

The value of an intervention window at each *tu z* to the rail manager (Eqn (3)), i.e. the value of being able to decide in *tu z* whether or not a renewal intervention is to be executed or nothing is to be done is given in Figure 2. For example, if the intervention window is in *tu* 46, the intervention window has a value of 69.42×10^6 *mus* (point A in Fig. 2). This value is representative of the additional profit that would be possible for the rail manager when compared the situation where there was no possibility to execute a renewal intervention for the entire 80 *tus*. The value of total revenue if do-nothing strategy is defined from beginning till the end of the investigated period is 170.23×10^6 *mus*, which is calculated as cumulative sum of the first two term in Eqn (1).

As can be seen from Figure 2, the maximum value of an intervention window is 69.42×10^6 *mus*, which occurs when the intervention window is in *tu* 46. The 69.42×10^6 *mus* is the difference between the expected benefits if there was no intervention window, and therefore no intervention would be executed for the entire 80 *tus*, and the expected benefits if the intervention window is in *tu* 46 and the rail manager makes the best decision at that time as to whether or not an intervention should be executed.

The optimal intervention window depends on the expected amount that can be charged for transporting goods. It can also be seen in Figure 2 that if the intervention window was moved earlier (e.g. to *tu* 20 (point C)) or later (e.g. to *tu* 65 (point B)) that the value of the intervention window would be less.

As the intervention window approaches $t = 0$, its value decreases because the increase in knowledge with respect to the price of transporting goods decreases. In other words, it is less and less likely that the price of transporting goods will be known with enough certainty to justify the execution of a rehabilitation intervention. Between $t = 0$ and $t = 10$ the value of the intervention

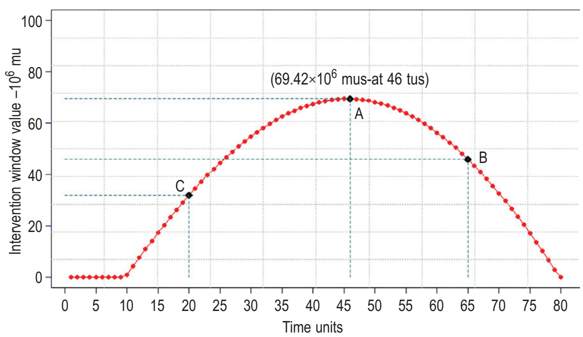

Fig. 2. Values of intervention windows

window is 0 (Fig. 2) due to the maximum sign in the third polynomial of the objective function (Eqn (1)), i.e. there is no value of transporting goods that will justify the execution of a rehabilitation intervention.

As the intervention window approaches T its value decreases because there is insufficient time in the investigated period to recoup the money spent on the rehabilitation intervention if executed.

4. Sensitivity analysis

The values of the discount factor ρ, parameters of geometric Brownian motion μ and standard deviation of the price of transporting good, σ, could have a significant effect on the value of the intervention window and therefore the optimal time to have an intervention window. The effect of variations in their values was investigated using the ranges of values shown in Table 4. The ranges selected were considered to be representative of the largest variations that one would expect in practice. The value and z of the optimal intervention window are shown in Figures 3, 4 and 5 for the different values of the discount factor, drift parameter and standard deviation, respectively.

Table 4. Ranges of values used in the sensitivity analysis

Parameter	Minimum		Maximum	
	Value	Percentage of initial estimate	Value	Percentage of initial estimate
ρ	1%	N/A	10%	N/A
μ	0.0001	–85	0.01	+85
σ	0.10	–70	0.60	+70

It can be seen from Figures 3–5, that the value of the intervention window and the optimal intervention window:

- Depend on the value of discount factor. The higher the value of the discount factor the higher the value of the intervention window and the earlier the optimal intervention window occurs (Fig. 3). For example, if the discount factor increases by 0.01, the value of the intervention window increases 5×10^6 *mus*, and if the discount factor increases by 0.02, the value of the optimal intervention window moves forward in time by approximately one *tu*. This is logical because as the value of the discount factor is increased the value of future benefits are decreased and, and therefore, it is better to have the intervention window earlier than later.
- Are not significantly dependent on the value of the drifting parameter, i.e. the base rate of increase in the price of transporting goods. The higher the value of the drifting parameter the higher the value of the optimal intervention window and the later the optimal intervention window occurs. These changes are, however, relatively small. As can be seen in Figure 4, if the value of the drifting parameter increases by 1×10^{-4} there is an increase in the value of the intervention window by approximately 1×10^6 *mus*, and the time of the optimal intervention window increases by only approximately one unit of time (e.g. one year). This is logical because as the rate of price increase increases it becomes increasingly beneficial to have the possibility to execute an intervention at a later time.
- Depend greatly on the standard deviation of the price of transporting goods (Fig. 5). The higher the value of the standard deviation the higher the value of the optimal intervention window and the later the

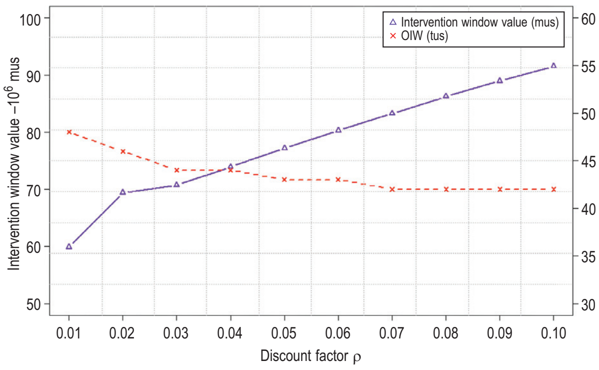

Fig. 3. Values of optimal intervention windows for different values of the discount factor

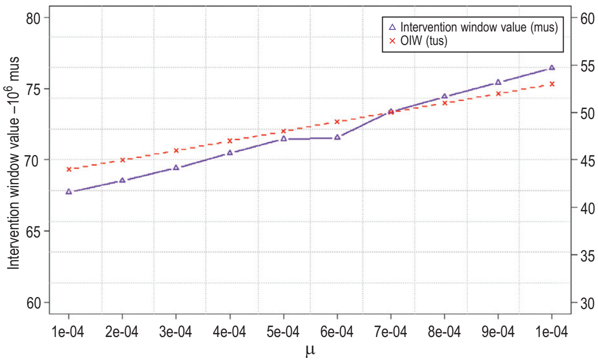

Fig. 4. Values of optimal intervention windows for different values of the drift parameter

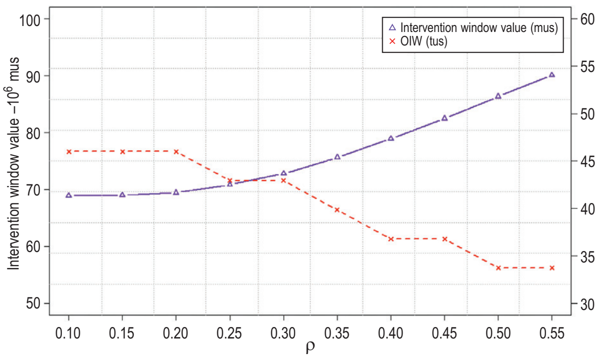

Fig. 5. Values of optimal intervention windows for different standard deviations of the price of transporting goods

optimal intervention window is. This is logical because as the standard deviation increases there is an increased uncertainty and, therefore, more value in waiting to determine whether or not to execute an intervention. This effect would continue so until one was sufficiently close to the end of the investigated time period that the benefits of executing an intervention would no longer outweigh the costs.

Conclusions

A real option approach to the determination of the optimal intervention window for a rail manager was presented. The approach is based on an adaptation of the Black and Scholes (1973) differential equation model used to value European call options in financial engineering. It is demonstrated by determining the optimal intervention window for a rail manager who manages infrastructure in a fictive rail corridor. The presented approach is one that would be useful in the management of rail infrastructure that belongs to a multi-national rail corridor where multiple railway organizations are responsible for maintenance. Once the value of all possible intervention windows for each rail manger were determined, the rail managers would be better positioned to negotiate with each other to select the intervention window(s) for the freight corridor. The selection of this intervention window by the rail managers would ensure that both interventions and train schedules could be better planned. The former would result in reduced costs by being able to combine interventions on multiple infrastructure objects. The latter would result in a reduction of disruption to users of the freight corridor. This is in line with Directive 2012/34/EU (2012) which aims to reduce problems associated with corrdination and establish a single European railway area.

As the selected intervention window would certainly not be the one that is optimal for all rail managers it is feasible that rail managers for which it is not optimal are, in some way, compensated for being forced to select a non-optimal intervention window. The amount of this compensation could be related to the difference between the exact interventions executed, and ones that would be executed if the rail manager was not constrained by the intervention window.

Future research work should be focused on applying this approach to a real world example, with particular focus on the administrative hurdles to be overcome, the complexity of dealing with many infrastructure objects that comprise a freight corridor, the complexity of train schedules and the determination of appropriate levels of compensation for rail managers who accept to execute interventions, which are for them, at non-optimal time.

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Appendix

The proof of Black and Scholes equation

The proof of Black and Scholes equation has been shown extensively in numbers of books on real option. For ease of understanding a brief description of the valuation of the European call option is given here. It is an abbreviated version of that given in Iacus (2011).

Let B_z be a standard Brownian motion and define B_z^x as Brownian motion that starts from x at time 0, which is given by:

$$B_z^x = x + B_z. \quad (A.1)$$

To obtain Brownian motion that starts at x at time z, i.e. translated Brownian motion is used:

$$B_T^{z,x} = x + B_T - B_z,\ T \geq z. \quad (A.2)$$

The translated geometric Brownian motion can then be defined as:

$$Z_T^{z,x} = x + \int_z^T \mu Z_u^{z,x} du + \int_z^T \sigma Z_u^{z,x} dB_u, \quad (A.3)$$

which is a geometric Brownian motion which is at x at time z.

The process $\left\{Z_T^{z,x}, T \geq z\right\}$ satisfies the stochastic differential equation:

$$dZ_T^{z,x} = \mu Z_T^{z,x} ds + \sigma Z_T^{z,x} dB_T, \quad (A.4)$$

with the following explicit solution:

$$Z_T^{z,x} = x \exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)(T - z) + \sigma\left(B_T - B_z\right)\right\}. \quad (A.5)$$

If $\sigma\left(B_T - B_z\right)$ is rewritten as:

$$\sigma\left(B_T - B_z\right) = \sigma\sqrt{T - z} \cdot Y, \quad (A.6)$$

with $Y = (B_T - B_z)/\sqrt{T - z} \approx \Phi(0,1)$.

Then $\mathbb{E}g(Z_T^{z,x})$ can be calculated as:

$$\mathbb{E}\left\{\max\left[Z_T^{z,x} - K, 0\right]\right\} = \mathbb{E}\left\{\max\left[e^{\ln Z_T^{z,x}}, 0\right]\right\} = \quad (A.7)$$

$$\mathbb{E}\left\{\max\left[e^{\ln x + (\mu - \frac{1}{2}\sigma^2)(T - z) + \sigma\sqrt{T - z} \cdot Y} - K, 0\right]\right\}. \quad (A.8)$$

The benefit (payoff of a European call option) is zero if $Z_T^{z,x}$ is lower than the value of K (strike value of the European call option) and hence the expected value above will be zero as well. If only the expected value of the positive trajectories are calculated, than:

$$\max\left[e^{\ln x + (\mu - \frac{1}{2}\sigma^2)(T - z) + \sigma\sqrt{T - z} \cdot Y} - K, 0\right] = 0. \quad (A.9)$$

If

$$e^{\ln x + (\mu - \frac{1}{2}\sigma^2)(T - z) + \sigma\sqrt{T - z} \cdot Y} \leq K = e^{\log K}, \quad (A.10)$$

or, better, if:

$$\log x + (\mu - \frac{1}{2}\sigma^2)(T - z) + \sigma\sqrt{T - z} \cdot Y < \log K, \quad (A.11)$$

then:

$$Y < \frac{\log K - \log x - (\mu - \frac{1}{2}\sigma^2)(T - z)}{\sigma\sqrt{T - z}}, \quad (A.12)$$

which can be rewritten as $Y < -d_2$, where:

$$d_2 = \frac{\ln\left(\frac{x}{K}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)(T-z)}{\sigma\sqrt{T-z}}. \qquad (A.13)$$

Thus:

$$\mathbb{E}\left\{\max\left[Z_T^{z,x} - K, 0\right]\right\} = \mathbb{E}\left\{Y1_{\{Y>-d_2\}}\right\} = \qquad (A.14)$$

$$\int_{-d_2}^{\infty}\left(e^{\ln x + (\mu - \frac{1}{2}\sigma^2)(T-z) + \sigma\sqrt{T-z}\cdot y} - K\right)\phi(y)dy$$

$$xe^{\mu(T-z)}\int_{-d_2}^{\infty} e^{-\frac{\sigma^2(T-z)}{2} + \sigma\sqrt{T-z}\cdot y}\phi(y)dy - K\int_{-d_2}^{\infty}\phi(y)dy,$$

where $\phi(y)$ is the density function of the standard Gaussian random variable, i.e $\phi(y) = e^{-\frac{y^2}{2}} / ^{-\frac{y^2}{2}} \sqrt{2\pi}\sqrt{2\pi}$. By symmetry of the Gaussian density:

$$\int_{-d2}^{\infty}\phi(y)dy = P(Y > -d_2) = P(Y < d_2) = \Phi)d_2) \qquad (A.15)$$

and then:

$$\mathbb{E}\left\{\max\left[Z_T^{z,x} - K, 0\right]\right\} =$$

$$xe^{\mu(T-z)}\int_{-d_2}^{\infty} e^{-\frac{\sigma^2(T-z)}{2} + \sigma\sqrt{T-z}\cdot y}\phi(y)dy - K\int_{-d_2}^{\infty}\phi(y)dy. \qquad (A.16)$$

If the variable of integration in the first integral is changed to $w = y - \delta\sqrt{T-z}$:

$$\int_{-d_2}^{\infty} e^{-\frac{\sigma^2(T-z)}{2} + \sigma\sqrt{T-z}\cdot y}\phi(y)dy. \qquad (A.17)$$

Then the following is obtained:

$$\int_{-d_2-\sigma\sqrt{T-z}}^{\infty} e^{-\frac{\sigma^2(T-z)}{2} + \sigma\sqrt{T-z}\cdot(w+\sigma\sqrt{T-z})}\frac{1}{\sqrt{2\pi}}e^{-\frac{(w+\sigma\sqrt{T-z})^2}{2}}dw;$$

$$\int_{-d_2-\sigma\sqrt{T-z}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\sigma^2(T-z) + \sigma\sqrt{T-z}\cdot w + \sigma^2(T-z) - \frac{1}{2}w^2 - \frac{1}{2}\sigma^2(T-z) - w\sigma\sqrt{T-z}}dw;$$

$$\int_{-d_2-\sigma\sqrt{T-z}}^{\infty}\frac{e^{-\frac{1}{2}w^2}}{\sqrt{2\pi}}dw; \qquad (A.18)$$

$$P\left(Y > -d_2 - \sigma\sqrt{T-z}\right) = P\left(Y < d2 + \sigma\sqrt{T-z}\right) = \Phi(d_1).$$

Together, Eqns (A.16) become:

$$\mathbb{E}\left\{\max\left[Z_T^{z,x} - K, 0\right]\right\} =$$

$$xe^{\mu(T-z)}\int_{-d_2}^{\infty} e^{-\frac{1}{2}\sigma^2(T-z) + \sigma\sqrt{T-z}\cdot y}\phi(y)dy - K\Phi(d_2)$$

$$x^{e^{\mu(T-z)}}\Phi(d_1) - K\Phi(d_2). \qquad (A.19)$$

Therefore, if that:

$$\Delta[z,x] = e^{-\rho(T-z)}\mathbb{E}f\left(Z_T^{z,x}\right), \qquad (A.20)$$

then:

$$\Delta[z,x] = x\Phi(d_1) - e^{-\rho(T-z)}K\Phi(d_2). \qquad (A.21)$$

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